Arc Midpoint Computation Welcomes Trig Interpretation

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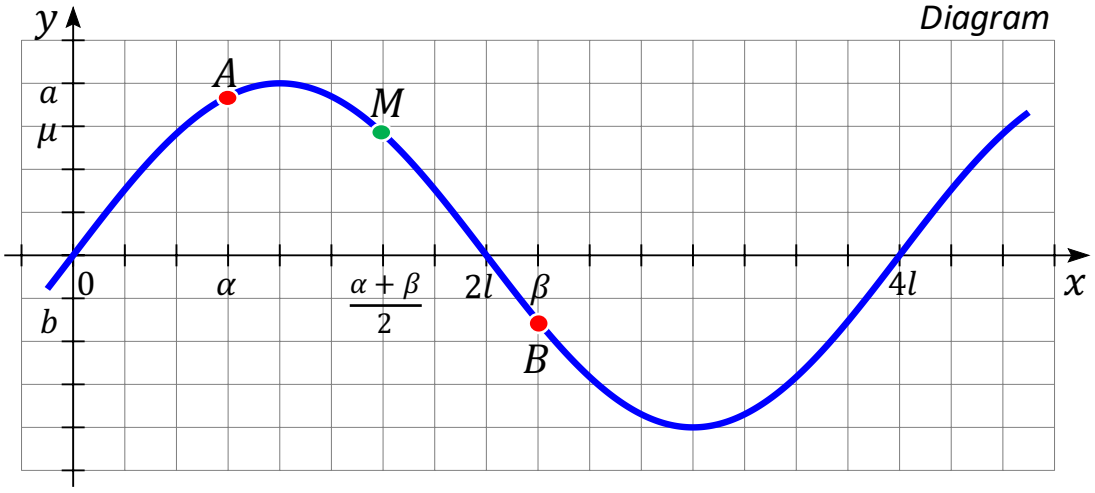
Sinusoidal functions and their graphs, described in Mathematics, are tightly related to the real processes involving repeating cycles, that are analyzed and imitated in Sciences. Sinusoid, a remarkable line of many amazing properties, models and visualizes this special type of wavelike relationships. Sine curve can be used to interpret wide spectrum of matters: the hours of daylight, the height of tides, transmission of sound, displacement of pendulum, circular motion, and it also gives the trigonometric interpretation of the arc midpoint computation. http://mathcentral.uregina.ca/RR/database/RR.09.10/akulov2.html

Consider sinusoid having period $4l$ and amplitude r. For its points $A(\alpha, a)$, $B(\beta, b)$ and $M\left(\frac{\alpha+\beta}{2}, \mu\right)$, $0 \le \alpha < \beta \le 4l$, see Diagram, show that

$$2\mu = \pm\sqrt{(r+a)(r+b)} \pm \sqrt{(r-a)(r-b)},$$

where the first radical has " $-$ " **iff** $3l \in [\alpha, \beta]$,
and the second radical has " $+$ " **iff** $l \in [\alpha, \beta]$.

Diagram



Example. If $r = 65$, $a = 56$, $b = -16$, for A and B located as shown,
$2\mu = \sqrt{(65+56)(65-16)} + \sqrt{(65-56)(65+16)}$, hence $\mu = 52$.