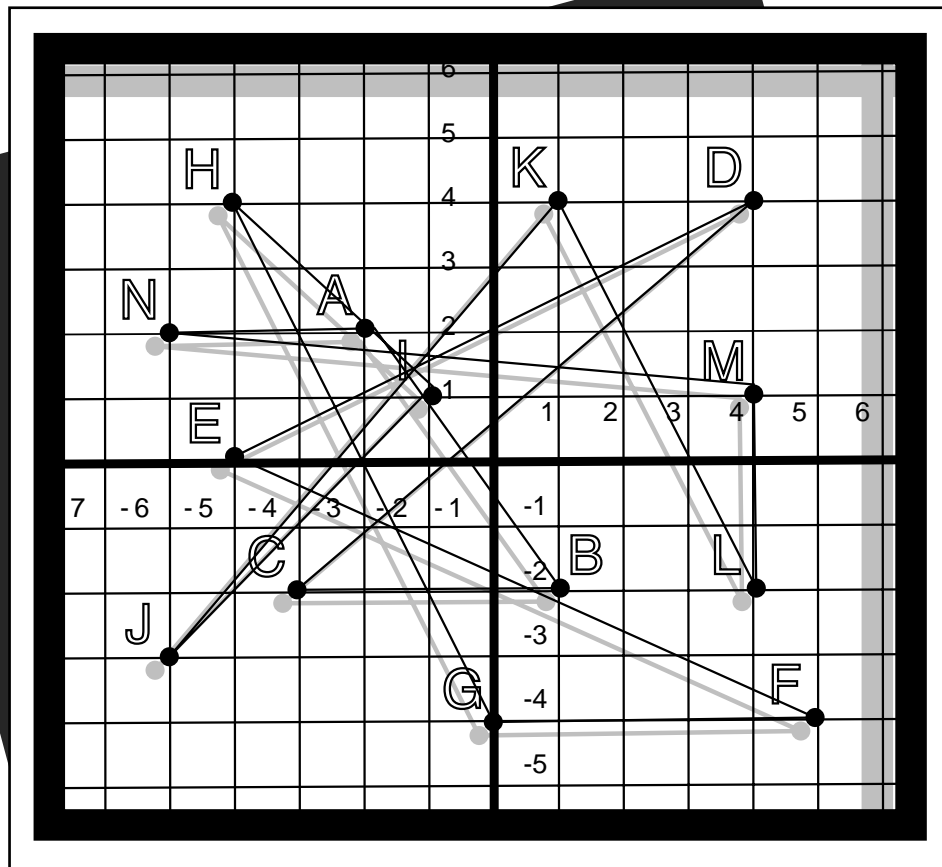


# Introduction to Slope for Middle Years Mathematics



by:  
Murray Bruce

1 9 9 4  
**S105.15**

TEACHING MATERIALS  
*from the*  
STEWART RESOURCES CENTRE





# INTRODUCTION TO SLOPE FOR MIDDLE YEARS MATHEMATICS

by Murray Bruce  
CELS  
1994  
S105.15

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## ANALYTIC GEOMETRY

### TOPIC:

### INTRODUCTION TO SLOPE FOR MIDDLE YEARS MATHEMATICS

#### Precis:

This topic deals with the Cartesian grid and linear equations with some analysis of the graph of the line and its relationship to the equation of the line. This topic was completed in five - fifty minute periods.

### I. BACKGROUND

Grades 1-5 Saskatchewan Curriculum

Geometry Strand: Topic-Plane

G-17 Recognize, Draw, Name:

a) point, line, line segment → Grade 3/4

b) parallel, intersecting and perpendicular lines → Grade 5

G-30 Locate and plot ordered number pairs in the first quadrant → Grade 5

Grades 7/8 Geometry

Coordinate grid-plotting points

Definitions of Euclidean concepts

### II. RESOURCES

- geoboard
- TI-81 Emulator Program (Macintosh Computer)
- coordinate grid (I used a coordinate grid which was a Clarisworks creation by Jonathan Elder, a Grade Eleven student at Fillmore)

### III. SEQUENCE

#### *Coordinate System*

- Grid Systems - examples
- Grid
- Locating Coordinates
  - examples
  - geoboard

#### Relations

##### Forms

- Mapping Notation
- Function Notation
- Equations of Lines

- $y = 2x$ ,  $y = 3x$ ,  $y = -2x$ ,  $y = -3x$  with  $x$ ,  $y$  charts.
- geoboard
  - computer (TI-81 Emulator program)

### Slopes of Lines

Use equations  $y = 2x$ ,  $y = 3x$ ,  $y = -2x$ ,  $y = -3x$  to explain  $m = (\text{rise})/(\text{run})$

- use  $y$ -intercept as a starting point
- positive, negative — horizontal and vertical and vertical slopes will be covered later.

$$y = + mx$$

$$y = + mx + b \text{ (slope-}y\text{-intercept equation)}$$

$$Ax + By = C$$

Calculating slopes using coordinates.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Horizontal and Vertical Lines

Parallel Lines-equations of lines used to illustrate the relationship between slope and parallel lines.

Intersecting Lines-equations of lines were used to indicate the relationship between slope and intersecting lines.

The students were instructed on use of the Emulator program and were to, with supervision, key in a couple of equations to graph. This topic took 4.5 classes of instruction with 0.5 of a class for an OPEN BOOK QUIZ.

## IV. STUDENT MATERIALS

- Graph Paper, Straight-Edge, Notebook

## V. CELs

- Communication, Creative and Critical Thinking, Numeracy, Technological Literacy

## VI. LESSONS

- with assignments
- instructional methods
- classroom management techniques

# LESSON 1

## PLOTTING POINTS ON A CARTESIAN GRID USING THE GEOBOARD

### LEARNING OBJECTIVE

The students will be able to plot and recognize points on a Cartesian coordinate grid. Use of a geoboard may be useful

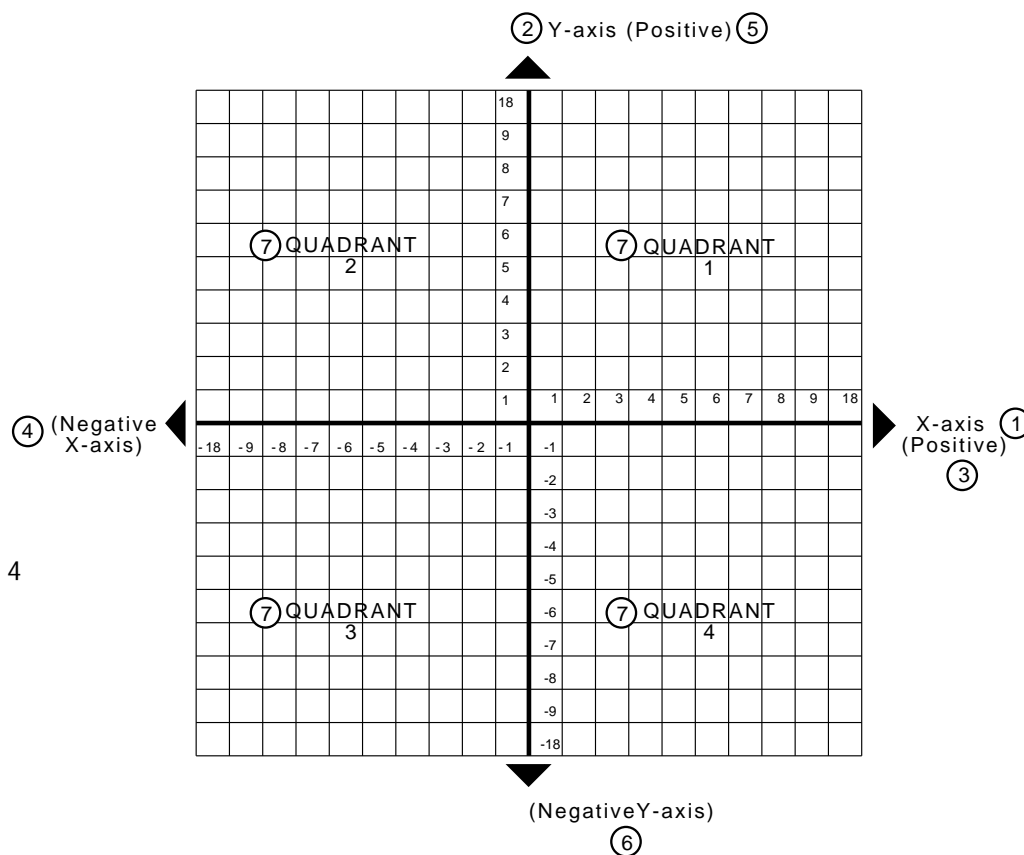
### CARTESIAN COORDINATE GRID (Descartes)

**Direct Instruction:** structured overview and demonstration of:

- Grid Systems
  - grid road maps - Saskatchewan
  - township maps or the local Rural Municipality Map
  - games - battleship
- Grid

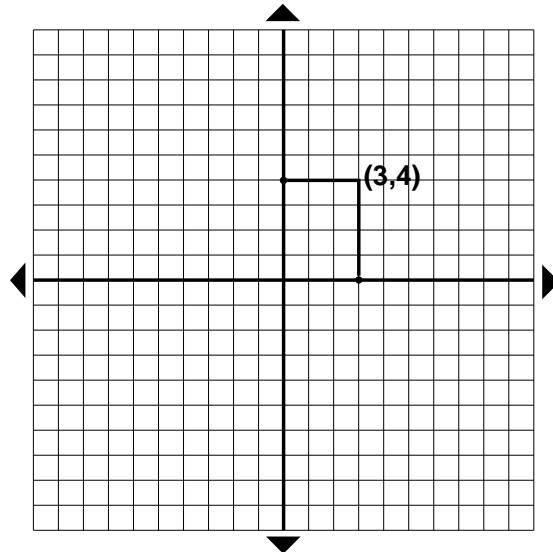
#### KEY:

1. x axis
2. y axis
3. positive x
4. negative x
5. positive y
6. negative y
7. quadrant 1, 2, 3, 4



# 4

- Locating Coordinates
  - example:  $(3,4)$  ———  $(x,y)$



- Geoboard
  - coordinate search (5 minutes)
  - given points A-K (oral correction with instructor writing correct answer on blackboard)
  - the instructor has previously placed the points on the geoboard using a nonpermanent marker.

## INTERACTIVE

peer practice (groups)

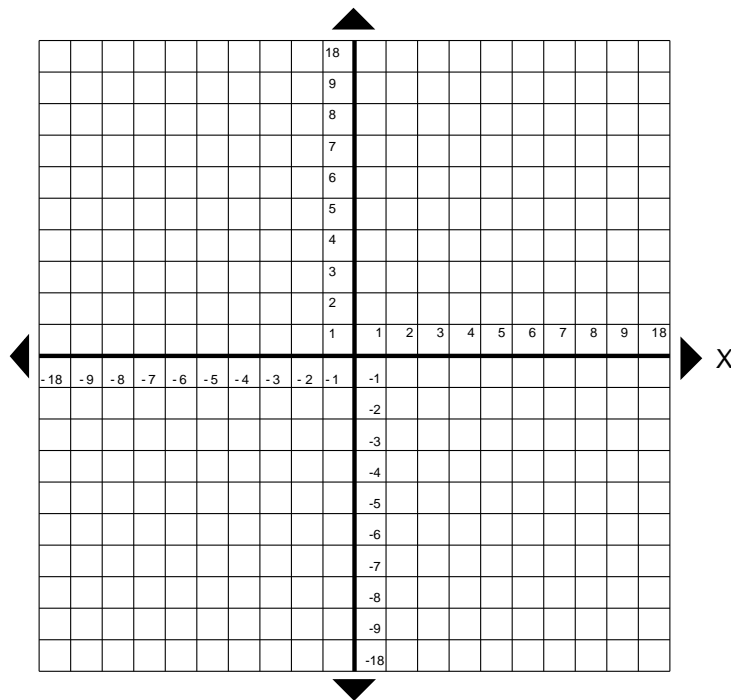


### CLASSROOM MANAGEMENT

Monitor use of nonpermanent markers when students plot points L, M, and N.



- A \_\_\_\_\_  
 B \_\_\_\_\_  
 C \_\_\_\_\_  
 D \_\_\_\_\_  
 E \_\_\_\_\_  
 F \_\_\_\_\_  
 G \_\_\_\_\_  
 H \_\_\_\_\_  
 I \_\_\_\_\_  
 J \_\_\_\_\_  
 K \_\_\_\_\_  
 L (4, -2)  
 M (4, 1)  
 N (-5, 2)



- plot 3 points L, M, N — hand out nonpermanent markers and when students are finished, take them back in.

## RELATIONS

**Direct Instruction:** explicit teaching of examples.

- a comparison of numbers: from newspapers, etc.

### Forms

Mapping Notation

x → 2x  
 -2 →  
 -1 →  
 0 →  
 1 →  
 2 →

Function Notation

$f(x) = 2x$

# 6

---

Equations of Lines (results will be as ordered pairs with x, y elements of the set of integers)

x, y charts

$$y = 2x$$

| x  | y |
|----|---|
| -2 |   |
| -1 |   |
| 0  |   |
| 1  |   |
| 2  |   |

$$y = 3x$$

| x  | y |
|----|---|
| -2 |   |
| -1 |   |
| 0  |   |
| 1  |   |
| 2  |   |

$$y = -2x$$

| x  | y |
|----|---|
| -2 |   |
| -1 |   |
| 0  |   |
| 1  |   |
| 2  |   |

$$y = -3x$$

| x  | y |
|----|---|
| -2 |   |
| -1 |   |
| 0  |   |
| 1  |   |
| 2  |   |

Results will be ordered pairs as  $y, y \in \mathbb{I}$

## ASSIGNMENT 1

- Plot points on graph paper (different grids)
- Label graph using equation.

## LESSON 2

### GRAPHING LINES ON THE CARTESIAN GRID

#### LEARNING OBJECTIVE

- The students will be able to graph line on a Cartesian coordinate grid.
- The students will be able to determine the slope and y-intercept of the line given:
  - the graph of the line
  - the equation in slope - y-intercept form

**Indirect Instruction:** reflective discussion on x, y charts using the set of integers.

#### CORRECT ASSIGNMENT ONE

Students graph on blackboard using different colored chalk (plot check).

- examine relationships between equation
  - positive and negative directions
  - origin

**Direct Instruction:** compare and contrast; demonstration

- use same equations where  $x, y \in \mathbb{R}$ 
  - geoboards - four elastics the students appreciated the use of the geoboards as it gave them an idea of the rise and the run.
  - TI-81 Mac Emulator program
  - brainstorm with students various examples of lines used in architecture, house design, natural landscapes (ski slopes), engineering designs — to develop an intuitive idea of the applications of lines.



#### CLASSROOM MANAGEMENT

first complete instructions on how to use equipment, e.g. elastics and geoboards and then pass equipment out.

## Slope

- Use equations from assignment 1 where  $x, y \in \mathbb{R}$ .

**Direct Instruction:** structured overview of:

- Slope
  - y-intercept equation:  $y = \pm mx$
  - use assignment 1 equations as examples.
  - the y-intercept is the origin.
- Slope
  - y-intercept equation:  $y = \pm mx + b$
  - the y-intercept is not the origin.
  - TI-81 Emulator program:  $y = mx + b$ 
    - instructor will key in sets of equations to illustrate different equations
  - the use of the TI-81 Emulator program by the students was somewhat reserved and they did not really understand the connection between the equation  $y = mx + b$  and the graph of the equation. This was corrected by some direct instruction on the blackboard and a paper and pencil assignment similar to the following:

**SET 1:**

- a)  $y = x + 1$
- b)  $y = x - 1$
- c)  $y = -x + 1$
- d)  $y = -x - 1$

examine similarities and differences.

**SET 2:**

- a)  $y = 2x - 1$
- b)  $y = 2x - 3$
- c)  $y = 2x + 1$
- d)  $y = 2x - 3$

examine similarities and differences.

## LESSON 3

### DETERMINING THE SLOPE AND Y-INTERCEPT OF THE LINE

#### LEARNING OBJECTIVE

Students will be able to determine the slope and y-intercept of the line given:

- the graph of the line (review)
- the equation in slope-y-intercept form (review)
- the equation in standard form

**Independent Learning:** computer assisted instruction

- Supervise students computer use - 0.5 hour
- Geoboard: use to illustrate the determination of slope using the y-intercept as a starting point.
- Slope:  $m = \text{RISE/RUN}$ 
  - use of elastics to determine the run from the y-intercept to the next x coordinate. Determine the rise.
- paper and pencil graphing to show how slope y-intercept graphing works, (as opposed to using an x, y chart).

**Direct Instruction:** structured overview and demonstration.

- Standard Form equations.
  - $Ax + By = C$ 
    - Solve for y:  $y = -(A/B)x + C/B$
    - From any given equation in standard form select the numerical coefficients for A, B and C and determine slope using  $m = \text{rise/run} = -A/B$ . For the y-intercept b use  $C/B$ .

**ASSIGNMENT 2**

1. Determine the slope and the y-intercept of the following equations:

a)  $y = x$

b)  $y = 7x$

c)  $y = -4x$

d)  $y = 2x + 7$

e)  $y = 8x + 6$

f)  $y = -4x^2$

g)  $y = -6 - 3x$

h)  $2x + y = 4$

i)  $7x + 6y = 4$

j)  $12y + 3x = 14$

2. Determine the equation of the line in:

i) slope - intercept form

a)  $m = 3$  and  $b = -2$

b)  $m = -7$  and  $b = 4$

c)  $b = 1$  and  $m = 3$

ii) standard form

d)  $-A/B = 6$  and  $C/B = 2$

e)  $-A/B = -3$  and  $C/B = 2$

f)  $-A/B = -1$  and  $C/B = 1$

The instructor and the students went through these questions as a group (except 1hij and 2def).

## LESSON 4

### DETERMINING THE SLOPE USING COORDINATE POINTS

#### LEARNING OBJECTIVE

- Students will be able to determine the slope of the line given two coordinate points.

**Direct Instruction:** structured overview

- Finish correcting Assignment 2
  - if any problems occur, use the geoboards or if time is a factor, use the blackboard or computer.
  - Calculating slopes using coordinates:
$$m = (y_2 - y_1)/(x_2 - x_1)$$
  - plot coordinates for some samples and then calculate the slope by substitution into the slope formula.

#### ASSIGNMENT 3

Determine the slope of:

AB where A(2,1) and B(-4,3)

CD where C(-3,5) and D(5,1)

EF where E(-11,5) and F(-5,7)

GH where G(-6,-5) and H(3,-1)

IJ where I(7,-5) and J(10,1)

KL where K(-3,8) and L(2,-2)

MN where M(3,3) and N(7,3)

OP where O(2,2) and P(2,4)

Indicate to students that the last two sets may have surprising results. Class time was given to work on the assignment and was corrected in the same class.

#### Horizontal and Vertical Lines

- the slopes of MN and OP were used to illustrate horizontal and vertical lines (graph of each was shown).
- several samples were given on the blackboard and equations were used to indicate connection with coordinates and equations.

**LESSON 5****PARALLEL AND INTERSECTING LINES IN RELATION TO SLOPE****LEARNING OBJECTIVE**

- Students will be able to determine whether lines are parallel or intersecting based on their slope given:
  - the graphs of the lines
  - the equations of the two lines in slope - y-intercept form.

**Direct Instruction:** structured overview and demonstration

- Parallel Lines
  - equations of lines were used to illustrate the relationship between slope and parallel lines.
- Intersecting Lines
  - equations of lines were used to indicate the relationship between slope and intersecting lines.

**VII. SUMMARY FOR STUDENTS** - review of learning objectives

**VIII. EVALUATION**

- QUIZ — OPEN BOOK — 20 MINUTES  
THE RATIONALE FOR THE OPEN BOOK NATURE WAS THAT THIS WAS THE FIRST VISUAL REPRESENTATION OF THE EQUATION OF THE LINE AS IT RELATES TO THE CARTESIAN GRID.



## ANALYTIC GEOMETRY

NAME \_\_\_\_\_

## OPEN BOOK QUIZ

/24

1. What are the coordinates for the points on the grid

A \_\_\_\_\_

B \_\_\_\_\_

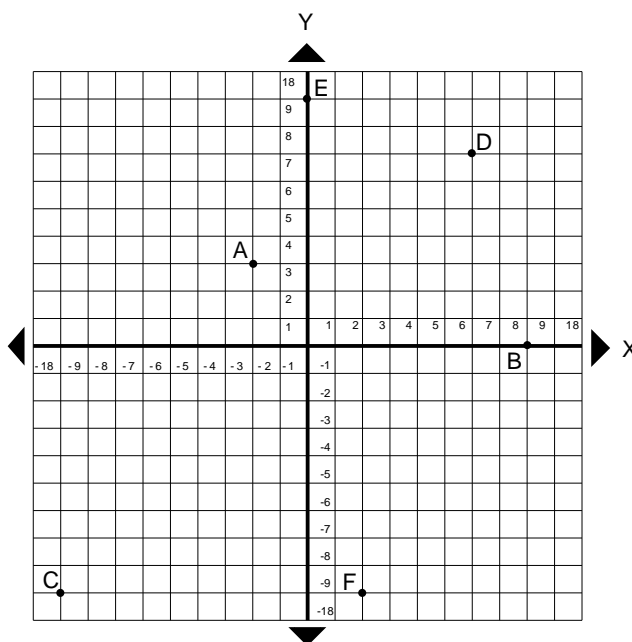
C \_\_\_\_\_

D \_\_\_\_\_

E \_\_\_\_\_

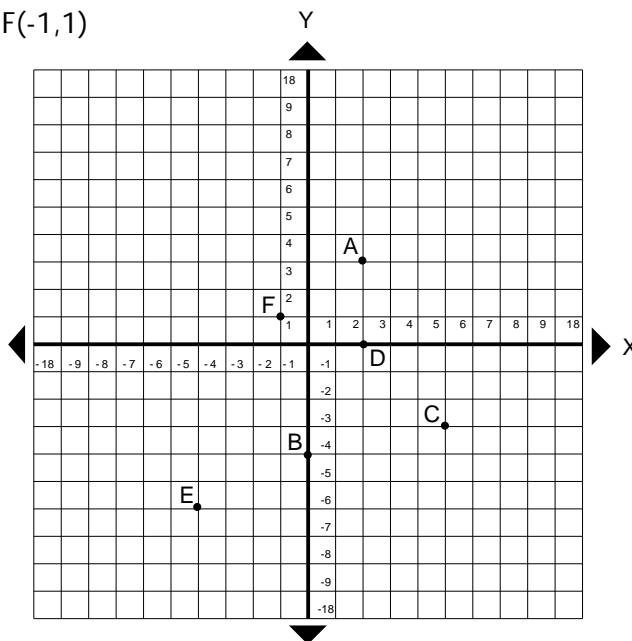
F \_\_\_\_\_

/3 marks



2. Plot the points on the grid:

A(2,3); B(0,4); C(5,-3); D(2,0); E(-4,-6), F(-1,1)



/3 marks

3. Complete the x, y chart for the following equations:

a)  $y = 2x + 1$

| x  | y |
|----|---|
| -2 |   |
| -1 |   |
| 0  |   |
| 1  |   |
| 2  |   |

b)  $y = -x + 2$

| x  | y |
|----|---|
| -2 |   |
| -1 |   |
| 0  |   |
| 1  |   |
| 2  |   |

/2 marks

4. What is the slope for the following equations:

a)  $y = 7x - 3$

b)  $y = -3x + 3$

c)  $3x + 2y = 5$

/3 marks

5. What is the slope and y-intercept of the following equations:

a)  $y = -2x$

b)  $y = 7x + 6$

/4 marks

6. Determine the slope:

a)  $(-3,6)$   $(-1,3)$

b)  $(7,0)$   $(2,1)$

c)  $(0,3)$   $(0,6)$

/6 marks

7. For the equation sets given determine which are parallel and which intersect:

a)  $y = 7x - 2$   
 $y = -7x - 2$

b)  $y = 3x + 2$   
 $y = 3x - 3$

c)  $y = x - 1$   
 $y = x + 1$

/3 marks

## ANALYTIC GEOMETRY

NAME Key

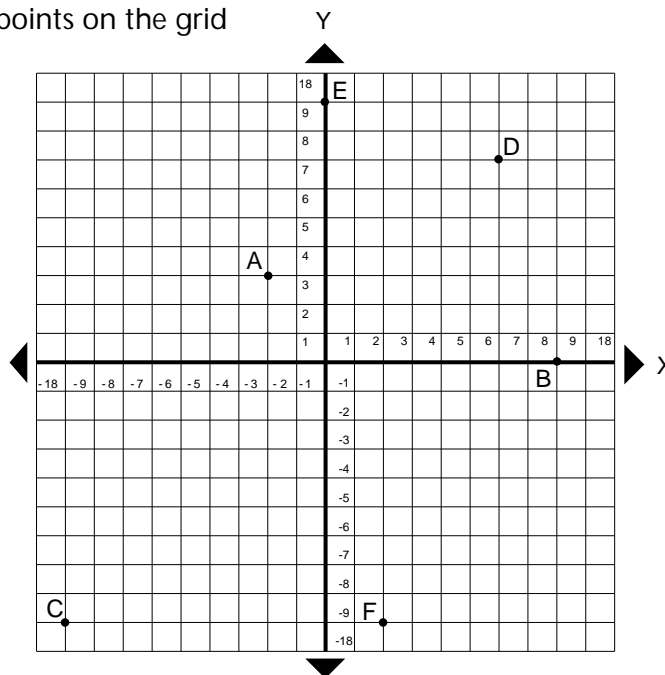
/24

## OPEN BOOK QUIZ

1. What are the coordinates for the points on the grid

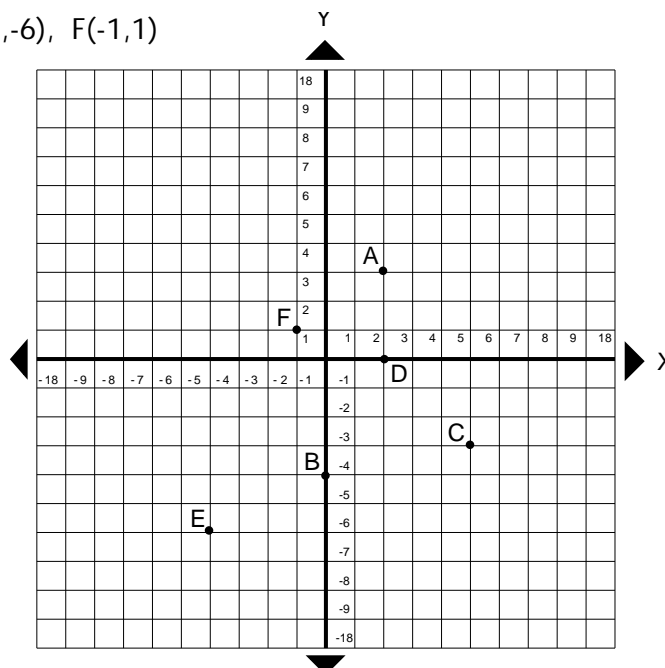
A  $(-2, 3)$ B  $(8, 0)$ C  $(-9, -9)$ D  $(6, 7)$ E  $(0, 9)$ F  $(2, -9)$ 

/3 marks



2. Plot the points on the grid:

A(2,3); B(0,4); C(5,-3); D(2,0); E(-4,-6), F(-1,1)



/3 marks

3. Complete the x, y chart for the following equations:

a)  $y = 2x + 1$

| x  | y  |
|----|----|
| -2 | -3 |
| -1 | -1 |
| 0  | 1  |
| 1  | 3  |
| 2  | 5  |

b)  $y = -x + 2$

| x  | y |
|----|---|
| -2 | 4 |
| -1 | 3 |
| 0  | 2 |
| 1  | 1 |
| 2  | 0 |

/2 marks

4. What is the slope for the following equations:

a)  $y = 7x - 3$

$m = 7$

b)  $y = -3x + 3$

$m = -3$

c)  $3x + 2y = 5$

$2y = -3x + 5$

$y = \frac{-3}{2}x + \frac{5}{2}$

$m = \frac{-3}{2}$

/3 marks

5. What is the slope and y-intercept of the following equations:

a)  $y = -2x$

$m = -2$

$b = -0$

b)  $y = 7x + 6$

$m = 7$

$b = 6$

/4 marks

6. Determine the slope:

a)  $(-3, 6)$   $(-1, 3)$

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 6}{-1 - (-3)} = \frac{-3}{2}$

b)  $(7, 0)$   $(2, 1)$

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 0}{2 - 7} = \frac{1}{-5}$

c)  $(0, 3)$   $(0, 6)$

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - 3}{0 - 0} = \frac{3}{0}$   
= undefined line

/6 marks

7. For the equation sets given determine which are parallel and which intersect:

a)  $y = 7x - 2$

$y = -7x - 2$

Intersect -

slopes are not equal

b)  $y = 3x + 2$

$y = 3x - 3$

Parallel -

slopes are equal

c)  $y = x - 1$

$y = x + 1$

Parallel -

slopes are equal

/3 marks