ANGLES AND POLYGONS

Mathematics 10
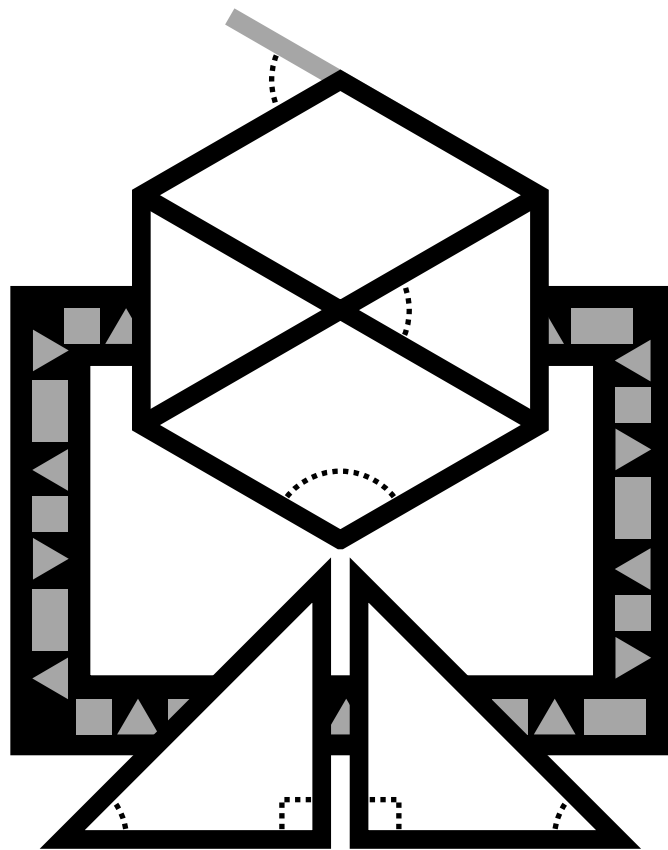
Submitted by:
Keith Seidler and Romesh Kachroo

INTRODUCTION

To meet a need for resources for the new Math 10 curriculum, the Saskatchewan Teachers' Federation, in cooperation with Saskatchewan Education, Training and Employment, initiated the development of teacher-prepared unit plans.

A group of teachers who had piloted the course in 1992-93 were invited to a two and a half day workshop in August, 1993 at the STF. The teachers worked alone or in pairs to develop a plan for a section of the course.

Jim Beamer, University of Saskatchewan, and Lyle Markowski, Saskatchewan Education, Training and Employment, acted as resource persons for the workshop.

TEACHER NOTE:

Throughout the sample units, the following symbols are used to refer to the Common Essential Learnings:

C	Communication
CCT	Critical and Creative Thinking
IL	Independent Learning
N	Numeracy
PSVS	Personal and Social Values and Skills
TL	Technological Literacy

SECTION 1

FOUNDATIONAL OBJECTIVES:

To identify and apply common properties of triangles, special quadrilaterals, and n-gons. (10 05 01). Supported by learning objectives 1, 2, 3, 4, 8, 9, and 10.

To apply the special quadrilaterals to real world situations. (10 05 02). Supported by learning objectives 6 and 7.

To develop an understanding of Pythagoras' Theorem, the primary trigonometric ratios, and their applications. (10 05 03). Supported by learning objectives 11 to 17.

OBJECTIVES:

1. To define and illustrate terms related to the study of angles.
2. To solve diagrammatic and word problems involving these angles.

BACKGROUND:

Most terms should be familiar to students from previous mathematics courses. Some explanation may be required for regular polygons.

TIME:

2 periods

INSTRUCTIONAL METHODS OR ACTIVITIES:

Note: Activity 2 could be done before or after Activity 1 or left out. Activity 3 is intended as an adaptive dimension.

1. Students may use the following activity sheet to review, define, and illustrate angular terms. Following this, the students may be assigned suitable questions involving these terms. (IL, N, CCT) Examples from Geometry - Jurgensen 1985 edition could include:

PAGE	SUITABLE QUESTIONS	
13	classroom exercises	1-38
14	written exercises	1-38
31	classroom exercises	1-18
32	written exercises	1-32 challenge 35-38

2. Students may be broken into groups of 3 or 4 in which they discuss occupations involving a knowledge of angles (see pages 120-121 of mathematical curriculum guide). Presentations may be as suggested or a guest speaker may be invited into class to discuss how his/her occupation involves angles or to demonstrate to students any special equipment that occupation may use to measure angles. (PSVS, C, CCT, N)

ADAPTIVE DIMENSION:

Adaptive Dimension Students may be broken into groups to develop board games which combine the above objectives. Such games may be designed as a treasure hunt or as saving your best friend from certain doom. In this game, students must solve word problems which involve angles - the solution to the problem tells them the angular direction they should move to advance. The distance of the move may be given or found be solving algebraic equations. This game could be adapted to include different types of angular measurement - e.g. N30E or headings of 120 degrees. (CCT, TL, N, PSVS)

ASSESSMENT:

- informal questioning
- anecdotal records

ACTIVITY SHEET #1 - *ANGLES*
(Use straight edge for diagrams)

Term	Illustration	Definition

• *ANGLE*

• *ACUTE ANGLE*

• *RIGHT ANGLE*

• *OBTUSE ANGLE*

• *STRAIGHT ANGLE*

• *REFLEX ANGLE*

Term	Illustration	Definition
• *COMPLEMENTARY ANGLES*		
• *SUPPLEMENTARY ANGLES*		
• *ADJACENT ANGLES*		
• *VERTICAL ANGLES*		
• *CONGRUENT ANGLES*		
• *CENTRAL ANGLE OF A REGULAR POLYGON*		

SECTION 2

CONCEPT:

Properties of polygons

OBJECTIVES: *(3, 4, 5, 6 of the Curriculum Guide)*

3. To define and illustrate terms related to polygons.
4. To classify quadrilaterals and polygons.
5. To construct types of parallelograms.
6. To state and apply properties of parallelograms.

TIME:

3 periods

INSTRUCTIONAL METHODS OR ACTIVITIES:

Note: The teacher may decide to use some or all of the following depending upon students.

1. The students may be grouped into 3 or 4 for the following concept mapping activity. Each group will require a large sheet of bristol board and an envelope containing two or three of each of the following polygons of varying sizes or shapes: triangles (scalene, isosceles, equilateral, acute, right, obtuse); quadrilaterals (with no sides congruent both convex and non-convex, trapezoids, isosceles trapezoids, parallelograms, rectangles, rhombuses and squares); pentagons, hexagons, and octagons (at least one of each of these should be regular).

    Students would be required to set up a classification system for all above shapes, using the properties of the shapes (see A-2.03 curriculum guide instructional notes page 122). When completed, each group should be requested to explain their classification system. Geometric properties may be summarized on board - consistencies and inconsistencies of each system may be discussed. (CCT, C ,PSVS,)

2. Students need to study the properties of quadrilaterals and more specifically parallelograms. Students may complete a chart such as the following or page 171 - Jurgensen. Encourage students to draw sketches to experiment with each characteristic, or use MIRA to test some. (IL, TL)

    Alternately use the chart on page 171, Jurgensen 1985 as an assessment chart.

3. The teacher may wish to use the following Venn diagram to help explain the properties of quadrilaterals from general to specific.

U= QUADRILATERALS — Convex, Kites, Trapezoids, Non-Convex, Rectangles, Parallelograms, Rhombus, Square
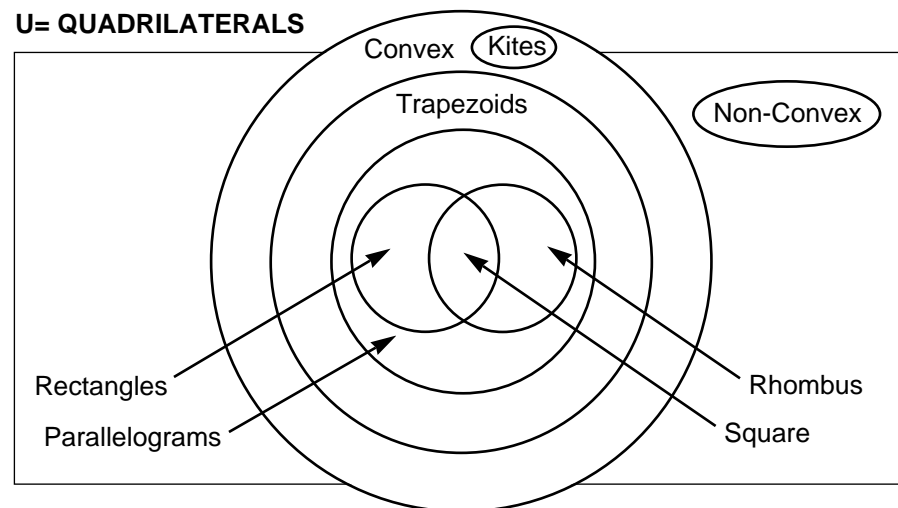
4. Student discussions could be used to explore methods of informally constructing parallelograms, rectangles, rhombuses, and squares. Paper folding and MIRA are options as suggested in the curriculum guide, page 122, as is the mechanical method using straight edge, rulers, and set squares. If formal constructions were covered previously (see page 118 of curriculum guide), then this may also be used. Students are encouraged to come up with other methods. (C, PSVS, CCT)

ADAPTIVE DIMENSION:

1. Students may be encouraged to search for examples of polygons in nature. For example, the hexagons of honeycombs, pentagons on shells of turtles, polygons on body of giraffes, etc. Polygons are also used in construction, machinery, jewelry, etc. Presentations may be made in the form of posters where diagrams may be hand-drawn or pictures from magazines or as oral presentations of applications of polygons in specific occupations. Alternatively, students may wish to create picture-art in which all elements of the diagram are made from polygons or compass. Students may be encouraged to find out how to construct kaleidoscopes. See media house catalogue for available videos.

ASSESSMENT:

- On-going student activity
- Activity or assessment sheet following
- Homework
- Presentation Concept mapping
    - group assessment
    - presentation assessment

ASSESSMENT WORKSHEET FOR QUIZ/ASSIGNMENT
PROPERTIES OF POLYGONS WORKSHEET

(Use straight edge for diagrams)

Term	Illustration	Definition and/or Properties and/or Characteristics
• POLYGON		
• REGULAR POLYGON		
• CONVEX POLYGON		
• NON-CONVEX POLYGON		
• TRIANGLE		
• ACUTE TRIANGLE		
• RIGHT TRIANGLE		
• SCALENE TRIANGLE		
• OBTUSE TRIANGLE		

Term	Illustration	Definition and/or Properties and/or Characteristics
• ISOSCELES TRIANGLE		
• EQUILATERAL TRIANGLE		
• QUADRILATERAL		
• PARALLELOGRAM		
• RECTANGLE		
• RHOMBUS		
• SQUARE		
• TRAPEZOID		
• ISOSCELES POLYGON		

SECTION 3

OBJECTIVES: *(7, 8, 9, 10 from Curriculum Guide)*

7. To determine the sum of the measures of the interior and exterior angles of a convex polygon of n sides.
8. To determine the measure of a central angle in a regular n-gon.
9. To determine the measures of the interior and exterior angles of regular n-gons.
10. To determine the number of diagonals in a polygon of n sides from each vertex.

BACKGROUND:

Students should be able to draw and/or identify central angles, exterior (external) angles, and interior angles of regular polygons.

EQUIPMENT NEEDED:

Protractor, straight edge

TIME:

2 or 3 periods

INSTRUCTIONAL METHOD AND/OR ACTIVITIES:

Note: The following exercises may be preceded by a discussion on polygons in general and extended to regular polygons. The teacher may show which properties are similar to both groups.

1. Students may work in groups of 2 to complete exercises 1 and 2 (attached), and prepare a report to the class. Alternatively, the teacher may have groups meet and debate their results before presentations are made to the larger body. (PSVS, C, CCT, N)
2. Exercise 3 may then be completed in groups or as the teacher feels appropriate. Students may require some help to fill in the row dealing with n-gons. (Exercise 3) With guided discussion, some students may develop the sum of interior angle formula. (IL, CCT, N, C)
3. Students will need more practice in questions related to above. Geometry Jurgensen 1985, pages 80-84. (IL, N)

ASSESSMENT:

- Presentation
- Group Assessment

EVALUATION:

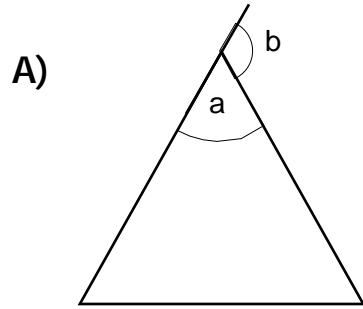The teacher may choose to administer a quiz or test at this point of this unit.

EXERCISE 1:

Draw the central angle ∠C and one external angle ∠E for each of the following regular polygons. Measure these and complete the table below. Write a short conclusion at the bottom of the page.

A)

B)

C)

D)
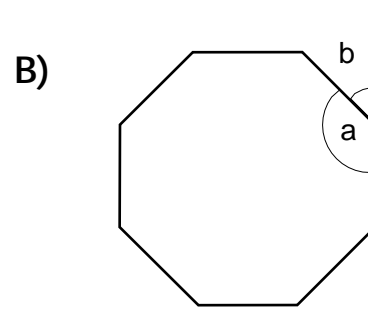
E)
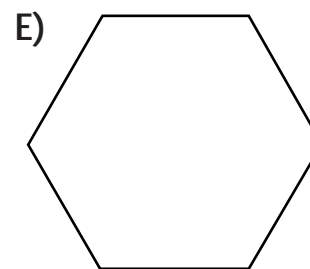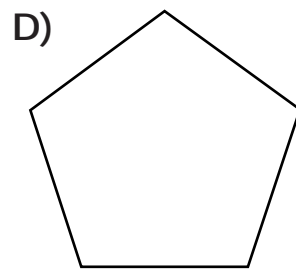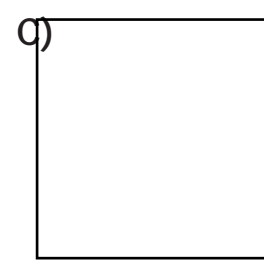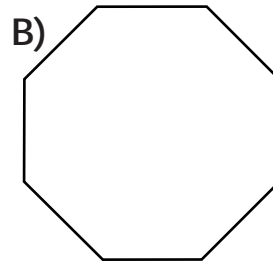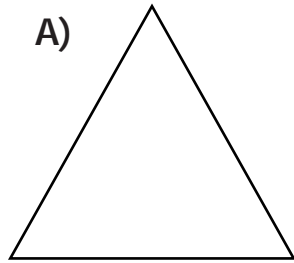
CONCLUSION:

	m∠C	m∠E
A		
B		
C		
D		
E		

EXERCISE 2:

Measure one interior angle (a) and one exterior angle (b) of each of the following regular polygons.

A)
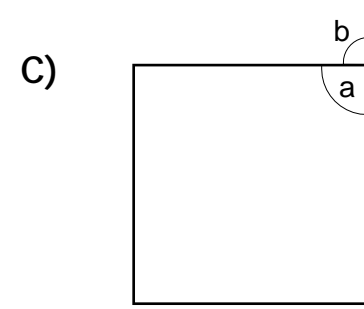
m∠ a =
m∠ b =
m∠ a + m∠ b =

B)

m∠ a =
m∠ b =
m∠ a + m∠ b =

C)

m∠ a =
m∠ b =
m∠ a + m∠ b =

D)

m∠ a =
m∠ b =
m∠ a + m∠ b =

E)

m∠ a =
m∠ b =
m∠ a + m∠ b =
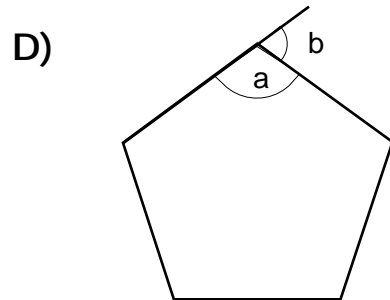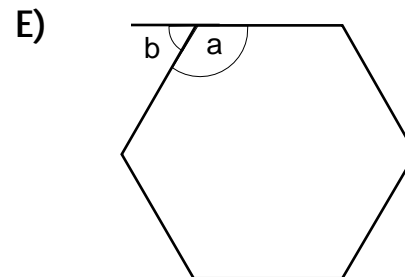
Since all the interior angles of a regular polygon are equal and all the exterior angles of a regular polygon are equals.

Find the sum of interior angles of:
A)
B)
C)
D)
E)

Find the sum of exterior angles of:
A)
B)
C)
D)
E)

11

EXERCISE 3:

number of sides	number of diagonals*	number of triangles	Sum of Interior Angles	m. of ext.∠'s of regular polygon	m. of int.∠'s of regular polygon	m. of central Angle	sum of all the exterior angles
3							
4							
5							
6							
7							
8							
9							
10							
•							
•							
•							
n							

KEY TO ABBREVIATIONS

* ....*From the same vertex*
m........*Measure*

SECTION 4

CONCEPT:
The Pythagorean Theorem

OBJECTIVES: *(11, 12, 13 from Curriculum Guide)*

11. To calculate to two decimal places the length of a missing side of a right triangle using the Pythagorean Theorem.
12. To solve word problems using the Pythagorean Theorem.
13. To determine of a triangle is a right triangle by using the converse of the Pythagorean Theorem.

BACKGROUND:
Knowledge of terminology of sides of right triangle; stress hypotenuse as the longest side, opposite the right angle; knowledge of solving simple equations, the meaning of the square root and use of a calculator to determine square roots.

TIME:
2 or 3 periods

INSTRUCTIONAL METHODS AND/OR ACTIVITIES:
Note: The teacher may want to bring in the historical perspective of the development of the Pythagorean Theorem. (Video "Donald in Mathmagic Land" by DISNEY may be fun)

1. The students may require some practice in determining the square root of teacher specified numbers using the calculator. (TL, N, IL)
2. The teacher may introduce the Pythagorean Theorem giving several examples on the board by altering the position of the right angle in the diagram. The teacher may have students estimate the value of the sides before determining the values using calculators. (CC, IL, N) (See sample pages on Pythagorean Theorem attached). Additional questions may be found in:
   Addison-Wesley, Math 10 (1987) - pages 380-391 Principles and Process 9 (1988) - pages 106-107 Geometry Jurgensen (1985) - pages 252-259 selective Math Is 4 - pages 37-42 Principles and Process 10 - pages 56-57
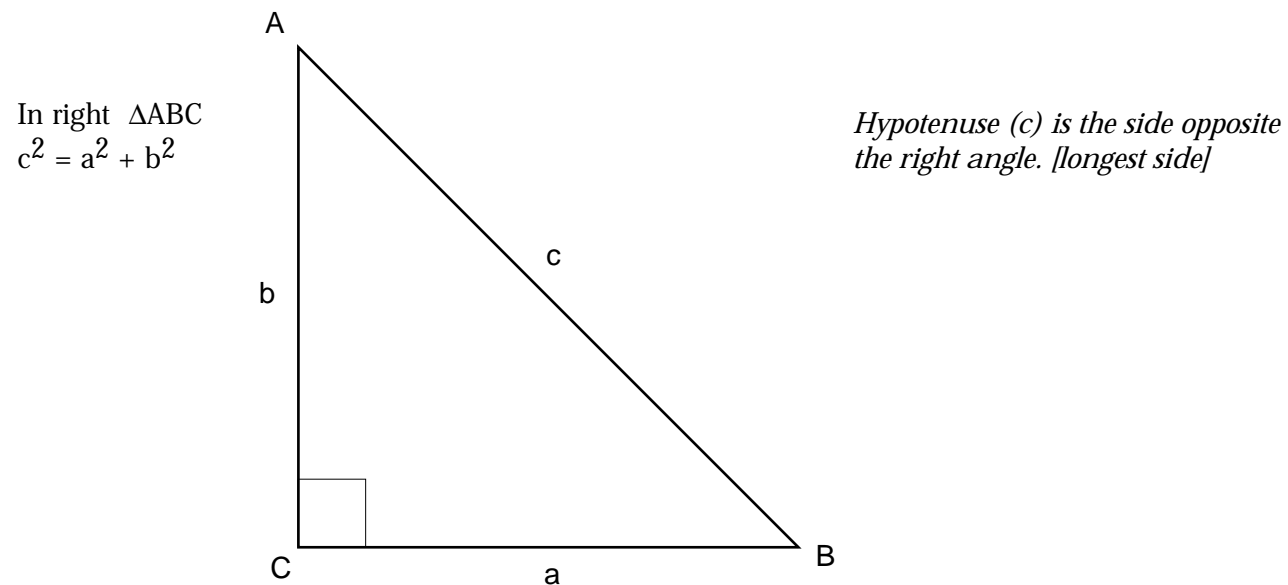
EVALUATION:
Quiz or Test

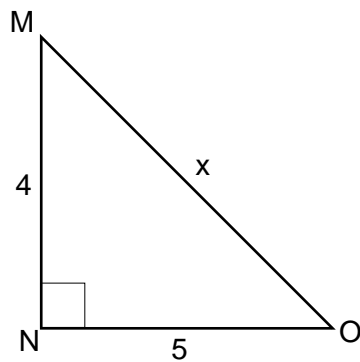## PYTHAGOREAN THEOREM:

The Pythagorean theorem states:

In a right triangle the square of the hypotenuse equals the sum of the squares of the other two sides.
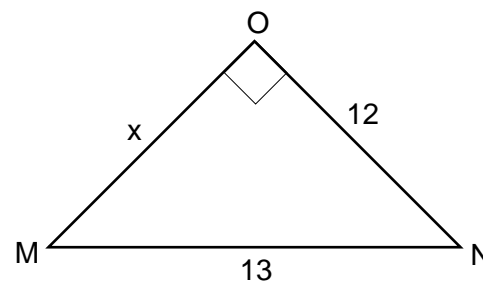

In right $\triangle ABC$
$c^2 = a^2 + b^2$

Hypotenuse (c) is the side opposite the right angle. [longest side]

What is the converse of this theorem?

EXAMPLES: *(On Board)*

Given a right triangle MNO find the value of x.

$MO^2 = MN^2 + NO^2$
$X^2 = 4^2 + 5^2$
$X^2 = 16 + 25$
$X^2 = 41$
$X = \sqrt{41}$
X = 6.4

$MN^2 = OM^2 + ON^2$
$13^2 = X^2 + 12^2$
$169 = X^2 + 144$
$169 - 144 = X^2$
$25 = X^2$
5 = X
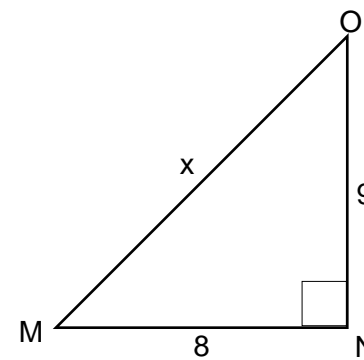
$OM^2 = MN^2 + NO^2$
$X^2 = 8^2 + 9^2$
$X^2 = 64 + 81$
$X^2 = 145$
X = 12.04

EXERCISES:

(Diagrams are not drawn to scale)

A) Find the indicated side for the following right Δ's.

i)

ii)

iii)

iv)

v)

vi)
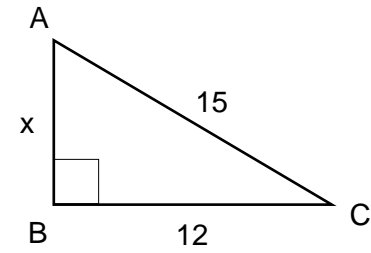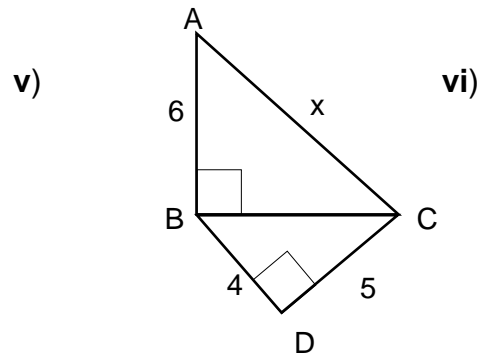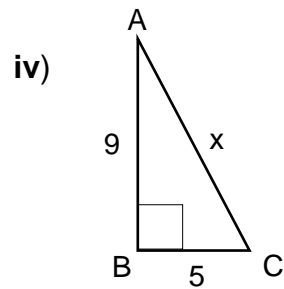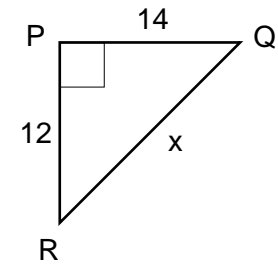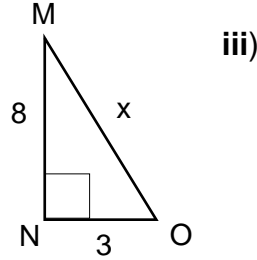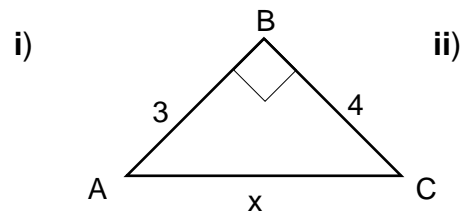
B) Which of the following are not right triangles? Give a reason.
(Note: Triangles may not be exactly as shown)

i)

ii)

iii)

15
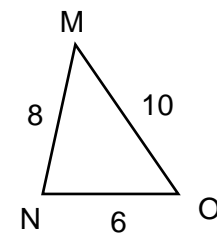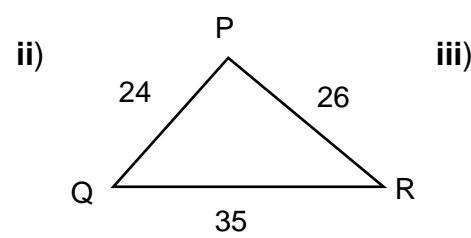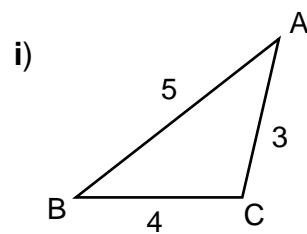
SECTION 5

ANGLES AND POLYGONS MATH 10

CONCEPT:

Trigonometric Ratios

OBJECTIVES: *(14, 15, 16, 17 from Curriculum Guide)*

14. To determine the value of the three primary trigonometric ratios using a calculator or tables.
15. To determine the measure of an angle given the value of one trigonometric ratio of the angle by using the calculator or tables.
16. To calculate the measure of an angle or the length of a side of a right triangle using the tangent, sine, and cosine ratios.
17. To solve word problems that involve trigonometric ratios using a calculator.

BACKGROUND:

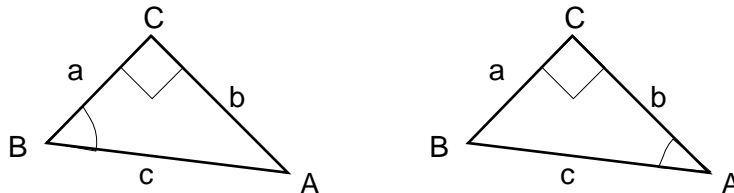Knowledge of right triangle and Pythagorean Theorem

TIME:

3 periods

INSTRUCTIONAL METHODS AND/OR ACTIVITIES:

1. The students may require practice in labeling the right angle, triangles in terms of hypotenuse, side adjacent and side opposite with reference to one of the two acute angles. (IL, CCT)

Example: $\triangle ABC$

In both diagrams the hypotenuse remains the same, however, the sides adjacent and opposite are interchanged when the reference angle is changed.

2. Students should become familiar with determining the sine, cosine, and tangent values by use of calculator (or tables if necessary). (TL, N, IL)
3. A more formal introduction of the trigonometric ratios may be used, in terms of hypotenuse, sides adjacent and opposite. A sample introduction on trigonometric ratios is included. Examples should lead into word problems and applications of these trigonometric ratios. Angles of evaluation and depression should be included. (CCT, C, N, IL, PSVS, TL)

Sample exercises may be found:
- Holt Math 10 (1987), pages 332-340
- Holt, Teachers Resource Manual (1987) - Master 13-1 and 13-2
- Math Is 4, pages 332-340
- Principles and Process 10 (1992), pages 462-480

4. The teacher may use the concepts of similar triangles when introducing trigonometric ratios.
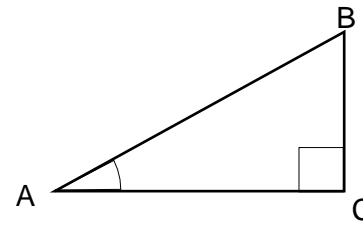
EVALUATION: *(Quiz on Trigonometric Ratios)*

Comprehensive Test on unit of Angles and Polygons may be given.

TRIGONOMETRIC RATIOS:

Sin function $\text{Sin A} = \dfrac{\text{opposite A}}{\text{hypotenuse}}$

Cos function $\text{Cos A} = \dfrac{\text{side adjacent A}}{\text{hypotenuse}}$

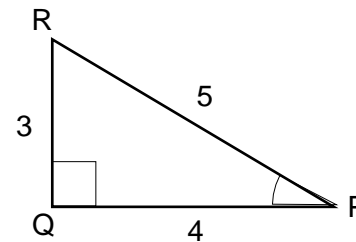Tan function $\text{Tan A} = \dfrac{\text{side opposite}}{\text{side adjacent}}$

EXAMPLE 1:

In ΔPQR let us find Sin P, Cos P, Tan P

$\text{Sin P} = \dfrac{\text{opposite A}}{\text{hypotenuse}} = \dfrac{3}{5}$

$\text{Cos P} = \dfrac{\text{side adjacent A}}{\text{hypotenuse}} = \dfrac{4}{5}$

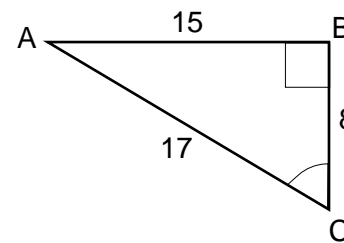$\text{Tan P} = \dfrac{\text{side opposite}}{\text{side adjacent}} = \dfrac{3}{4}$

EXAMPLE 2:

In ΔABC

Sin C =

Cos C =

Tan C =

UNIT ANGLES AND POLYGONS REFERENCES:

1. Geometry, Jurgensen, et all, 1985, Houghton Mifflin.
2. Geometry, Clemens, et al, 1984, Addison-Wesley Publishing Co.
3. Principles and Process 9, Ebos, et al, 1988, Nelson.
4. Principles and Process 10, Ebos, et al, 1992, Nelson.
5. Holt Math 10, Bye, Dale, et al, 1987, Holt, Rinehart and Winston.
6. Holt Math 10, Teachers Resource Manual, 1987, Holt, Rinehart and Winston.
7. Math Is 4, Ebos & Tuck, 1979, Nelson.
8. Math 10, Addison-Wesley (1987).