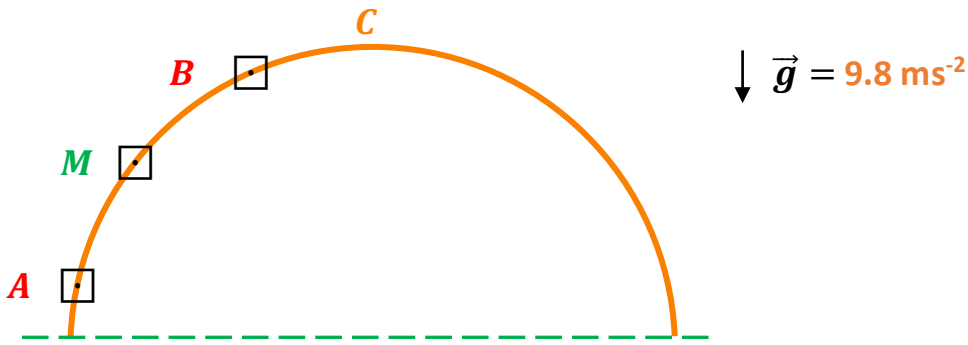
Arc Midpoint Computation Amplified by … Gravitation

Gregory V. Akulov, teacher, Luther College High School, Regina, Saskatchewan
Oleksandr G. Akulov, MM in OR, BMath, Vancouver, British Columbia

An object, that remains on a vertical semicircle having a horizontal diameter, changes its position along the semicircle. At the points ***A***, ***B***, ***C***, *and* ***M***, the object's gravitational potential energy in joules, relative to the diameter, is $a = 13\ \text{J}$, $b = 77\ \text{J}$, $c = 85\ \text{J}$, and $\mu\ \text{J}$, respectively. If ***C*** is highest point on the semicircle, and ***M*** is equidistant from ***A*** and ***B***, determine the **exact value** of μ.



The arc midpoint computation[1] approach to solving this problem gives the following result

$$2\mu = \sqrt{(85\ \text{J} + 13\ \text{J})(85\ \text{J} + 77\ \text{J})} - \sqrt{(85\ \text{J} - 13\ \text{J})(85\ \text{J} - 77\ \text{J})}\ , \text{ and } \mu = 51\ \text{J}\ .$$

1. Verify the answer using alternative approach. **Compare** solutions.
2. Show that the values of the **gravitational potential energy** a, b, c, and μ satisfy

$$2\mu = \sqrt{(c + a)(c + b)} \pm \sqrt{(c - a)(c - b)} \qquad (1)$$

3. Specify when (1) requires the sum of radicals, and when it requires their difference.
4. Modify (1) for the case when the entire circle instead of the semicircle is considered in the above.

[1] http://mathcentral.uregina.ca/RR/database/RR.09.10/akulov2.html