

A geometric view of the square root algorithm

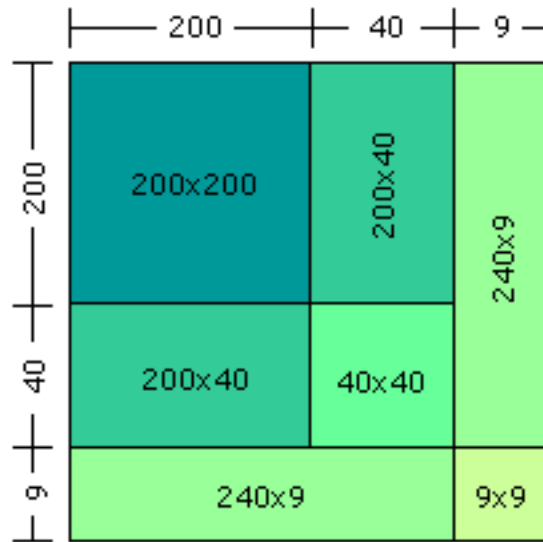
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There are two facts concerning the square of an integer that are useful in the inverse process of finding the square root. The first concerns the number of digits. In short:

If $0 < a < 10$ then $0 < a^2 < 100$,
if $10 < a < 100$ then $100 < a^2 < 10\,000$,
if $100 < a < 1000$ then $10\,000 < a^2 < 1\,000\,000$,
.
.
.

The point to see here is that the square of an integer has either twice as many digits as the integer itself, or one less than twice as many. So, since 9,409 has 4 digits its square root has 2 digits, and the square root of the 13 digit number 3,871,696,594,290 has 7 digits.

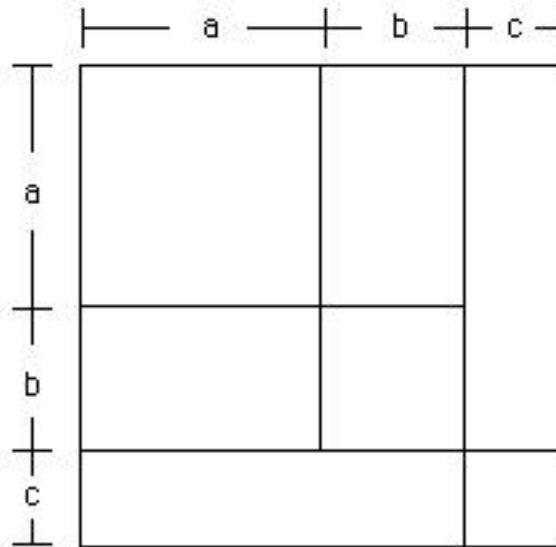
The second fact concerns the "geometry" of squaring a number. Consider, for example, the square of 249 so the geometric object to consider is a square with side of length 249 units. Write 249 as $200 + 40 + 9$ then the square can be seen as



and the square of 249 is $200 \times 200 + 2(200 \times 40) + 40 \times 40 + 2(240 \times 9) + 9 \times 9 = 62001$.

Now to find the square root of an integer you need first to determine the number of digits there will be. Let us find the square root of 64 009. Since 64 009 has 5 digits, the square root of $6 \mid 40 \mid 09$ will have 3 digits. First recall some of the geometric detail of a 3 digit number.

A square of a 3 digit number is divided into 7 parts as shown in the diagram.



$$\begin{array}{r}
 253 \\
 \hline
 200^2 \quad 64\,009 \\
 \underline{40\,000} \\
 24\,009 \\
 2 \times 200 \times 50 \quad \underline{20\,000} \\
 4\,009 \\
 50^2 \quad \underline{2\,500} \\
 1\,509 \\
 2 \times 250 \times 3 \quad \underline{1\,500} \\
 9 \\
 3^2 \quad \underline{9} \\
 0
 \end{array}$$

To find the square root of 64 009 we will start with the group of numbers furthest to the left. We will therefore start with the group whose only member is 6. This represents the square of a . We know that the largest perfect square less than 6 is 4, and that the square root of 4 is 2. Since 2 must be placed in the hundreds position, we know that $a = 200$. The area of region 1 is $200 \times 200 = 40\,000$. We then subtract this area from the total area of 64 009. By looking at the diagram we realize that next we should remove the areas of the two regions whose sides are a and b . To find the length of b we must estimate the quotient of 24 009 by 400. The 400 is arrived at by recalling that two regions, each of which has a length of 200, would have an overall length of $2(200) = 400$.

The quotient of 24 009 by 400 is approximately 60. However, looking again at our diagram we realize that besides the two areas of $a \times b$, we also must subtract an area of $b \times b$. Since $2 \times 200 \times 60 = 24\,000$ we are left with only 9, but we need to subtract $b \times b$ which is $60 \times 60 = 3\,600$. Thus we reduce our estimate of b to 50, and place a 5 in the tens position of the square root calculation. Two rectangles, each 200 units by 50 units, have a total area of 20 000 square units. Subtracting 20 000 from 24 009 leaves 4 009. Again, going back to the diagram we note that the area of $b \times b$ that must be subtracted is now $50 \times 50 = 2\,500$. Subtracting 2 500 from 4 009 leaves 1 509.

Returning to the diagram we note that next we must subtract the areas of the two regions which has a length of $a + b = 250$ units each. The unknown quantity c can now be estimated by the quotient of 1 509 by 500. 500 is arrived at by placing the two regions together to arrive at a rectangle with length $2 \times 250 = 500$. The quotient of 1 509 by 500 is approximately 3. Place 3 in the units position of the square root calculation and subtract the sum of the areas of regions 5 and 6, which is $2 \times 250 \times 3 = 1\,500$. Subtracting 1 500 from 1 509 leaves 9. From the diagram the region 7 is the only region not subtracted so far and its area is $c \times c = 3 \times 3 = 9$. Subtracting 9 from our previous remainder leaves us with a remainder of 0. Thus the square root of 64 009 is 253.