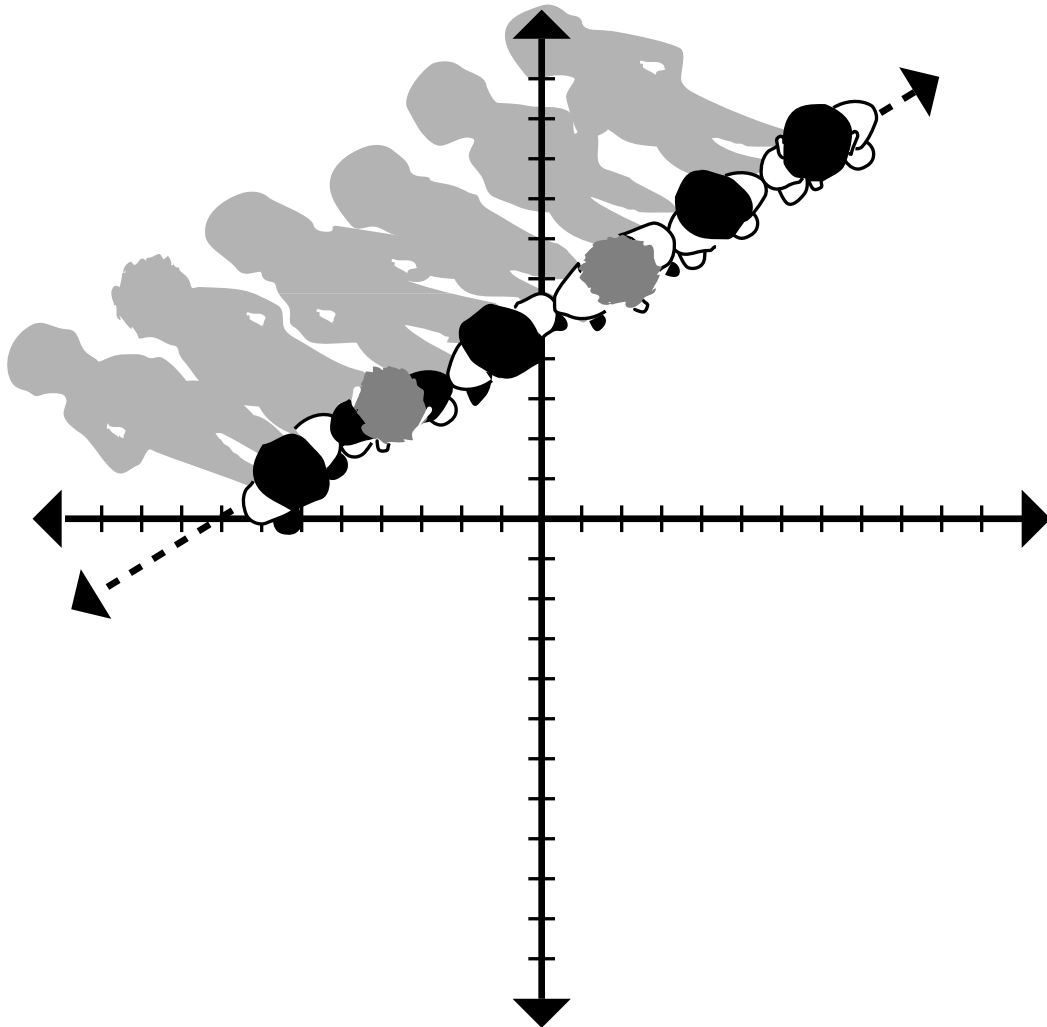


GET INTO LINE!



Math 10

“Core course unit on Linear Relations”



Submitted by: *Gale Russell*

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S105.8

TEACHING MATERIALS
from the
STEWART RESOURCES CENTRE



GET INTO LINE!

By Gale Russell

CELS

1993

S105.8

To meet a need for resources for the new Math 10 curriculum, the Saskatchewan Teachers' Federation in cooperation with Saskatchewan Education Training and Employment, initiated the development of teacher-prepared unit plans.

A group of teachers who had piloted the course in 1992-93 were invited to a two and a half day workshop in August, 1993 at the STF. The teachers worked alone or in pairs to develop a plan for a section of the course.

Jim Beamer, University of Saskatchewan and Lyle Markowski, Saskatchewan Education Training and Employment acted as resource persons for the workshop.

Get Into Line!

This unit is designed for the Math 10 Core course unit on Linear Relations. It covers seven sub-units dealing with the basics of graphing, the graphs of linear equations, the characteristics of a line, the writing of linear equations, and three areas of applications: direct and partial variation, arithmetic sequences and series, and scatterplots.

Most of the unit encourages the students' exploration of the topics without direct instruction, followed by practice questions and the students' own creation of problems. The unit attempts to tie the many different concepts together in order to give the students a more well-rounded perspective on linear relations.

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• SUB-UNIT 1: THE BASIC OF GRAPHING

LESSON 1: BASICS OF GRAPHING

Overview of Topics:

1. Review of graphing on a coordinate plane with extension to a four quadrant system.
2. Introduction to the terminology of graphing.
3. Practice in plotting and reading points on a graph.

Teacher Required Materials:

- Road map
- Copies of “The Spider and the Fly” for students (or similar exercise)
- *Chemistry Experiments and Principles*

Student Required Materials:

- Watch with a second hand (or clock in room)
- Graph paper
- Straight edge

Procedures/Activities:

1. To review what the students already know about graphing, have the students locate places by using the given coordinates. If the map uses numbers and letters, a discussion of the suitability of this type of system for mathematical purposes may prove interesting. The students will also have ideas from science which can be discussed.

Discuss physical situations where negative numbers may be encountered for example, look at the volume of a compound with respect to temperature changes (Boyle’s Law — see page 71 of *Chemistry Experiments and Principles* for a table of values that can be used. This discussion should lead to the development of the four quadrant system.

2. Define for the students: ordered pair, abscissa, ordinate, and axes. Also, it’s beneficial to discuss independent and dependent variables. In the previous example, if the temperature is changed, the volume will also change. Since we select the temperature, but measure the volume, the temperature is the independent variable and should be assigned to the horizontal axis.
3. As a graphing practice, give the students a puzzle to plot. (see appendix “The Spider and the Fly” — message: “Lunch at 6:10”)
4. To further practice the graphing of points, and of reading points, have each student plot a picture, message, or design with a minimum of 10 points, and then record the points as ordered pairs. (All four quadrants should be used). The student should then get another student to plot the points to see if the same result is achieved. The original puzzle, list of ordered pairs, and the trial plot can be turned in for evaluation. (See appendix: Creative Plotting)

Note: This activity should be done so that its results can be analyzed by the teacher before the next lesson

5. Have the students divide into pairs or groups of threes. Each student will assume a role as snapper, counter, or timer. (In pairs, snapper and counter can be combined). As the timer times out periods of 5 to 30 seconds at 5 second intervals, the snapper is to snap their fingers as many times as possible while keeping a steady pace. It is the counter's job to record the number of snaps in each time period in a table like the one below.

Time Interval (Seconds)	Number of Snaps
5	
10	
15	
20	
25	
30	

Before plotting the data, encourage the students to discuss which is the independent variable. Have the students turn in their graphs. Although the quadrants need not be the same size, encourage the students to show the four quadrants. See appendix for an assignment sheet.

Teacher Homework:

For each group graph, copy a grid onto an overhead sheet. Also, find the equation of the best fit line for their data and record it on the overhead sheet. This sheet, along with their original graph should be returned to the students at the beginning of the following lesson.

CELS:

- Personal and Social Value Skills, Communication, Creative and Critical Thinking

Evaluation:

- Rating scale on positive participation0 – 5
- Marked graphing project.....25

Time Line:

- 1 - 2 hours

• SUB-UNIT 2: GRAPHS OF LINEAR EQUATIONS

LESSON 2: LINEAR EQUATIONS TO LINEAR GRAPHS

Overview of Topics:

1. Plotting first degree equations from a table of values.
2. Recognizing a first degree equation and its relationship to a straight-line graph.

Teacher Required Materials:

- Overhead sheets and group graphs prepared from the last lesson
- Large graphing grid for demonstration purposes

Student Required Materials:

- *Mathematics 10*. Brendan Kelly, Paul Atkinson, and Bob Alexander. Don Mills, ON: Addison Wesley, 1987 (or other text selected by the teacher).
- Graph paper
- Straight-edge

Procedures/Activities:

1. Select an equation, such as $y = x - 3$, and have the students list possible things they could be asked to do with the equation. From previous experience, they should be able to identify substituting a value for a variable. Have the students give an example. Probe for why they picked the value and variable they did. Ask if they can make other substitutions. Encourage the use of a variety of numbers and for both variables. In each example, list $x = \underline{\hspace{2cm}}$, $y = \underline{\hspace{2cm}}$. Ask for ways to organize the results from each example into a cohesive grouping — lead into a table of values and sets of ordered pairs:

Example:

x	y	
-3	-6	$(-3, -6)$
0	-3	$(0, -3)$

x	y	
x_1	y_1	$\longrightarrow (x_1, y_1)$
x_2	y_2	$\longrightarrow (x_2, y_2)$
x_3	y_3	$\longrightarrow (x_3, y_3)$
:	:	

Ask what can now be done with this data —lead into a graph. Suggest evaluating intermediate values for x and y to see what happens between the points chosen. Encourage the students to discuss the pattern they see and make hypotheses regarding other values for the variables. Eventually, they should see that they get a line and that it continues on beyond the boundary of the graph paper. Brainstorm for ideas on how they might illustrate this type of continuation on their graphs. (This will help the students to see how mathematics notation is not necessarily magic, but the result of historical selections of ideas.)

2. Have the students reform their groups from last day and return their graphs with the overhead sheet with the equation you have prepared. Have the students create a table of values for their equation, being sure to use positive and negative values (each group member can contribute two points) and then graph it on the overhead sheet. They can then match up the origin of the original and new graphs along with the axes. The group should then discuss what they see and what it tells them about graphs and equations. The groups can then report their findings to the class on the relationship between graphs and equations.

3. To practice graphing using a table of values, and to develop an understanding of the relationship between linear equations and linear graphs, have the student do some questions from page 166 in *Mathematics 10*. It is good to do questions that are strictly equations, and those that have equations given within a word problem. Also have the students graph a few non-linear equations such as:

$y = x^2$		$y = x^3$		$y^2 = x$	
x	y	x	y	x	y
-2		-2			-2
-1		-1			-1
0		0			0
1		1			1
2		2			2

The students should then answer the following questions:

1. How are the last three graphs different from the previous graphs?
2. How are the last three equations different from the equations of the previous graphs?
3. Make a hypothesis about the relationship between the shape of a graph and its defining equation?

These three questions can either be discussed only as a class, or also turned in for evaluation.

CELS:

- Creative and Critical Thinking

Evaluation:

- Rating scale on positive participation0 - 5
- Homework completion rating scale.....0 - 2
- Plotting equations by tables of values quiz7

Time Line:

1 - 2 hours

• SUB-UNIT 3: CHARACTERISTICS OF A LINE

LESSON 3: INTERCEPTS AND SLOPE

Overview of Topics:

1. Finding the x and y intercepts.
2. Finding the slope of a graph from two points.

Student Required Materials:

- Graph paper
- Straight-edge
- *Mathematics: Principles and Process* (10). Frank Ebos et al. Scarborough, ON: Nelson Canada, 1990

Procedures/Activities:

- 1 Present the following problem to the class: When you arrive home from school one day, you realize that you do not have your house key, however, you can get into the garage where there is an extension ladder stored. You notice that the top window is open, so despite your fear of heights, you proceed to get the ladder.

What are some of the factors that become important when you set up the ladder?

Look for ideas about the angle at which you place the ladder, where on the ground it should start, where it needs to end, etc.

2. Next ask the students how the ladder situation is like the graph of a linear equation. Have them sketch different possible placements of the ladder (line).

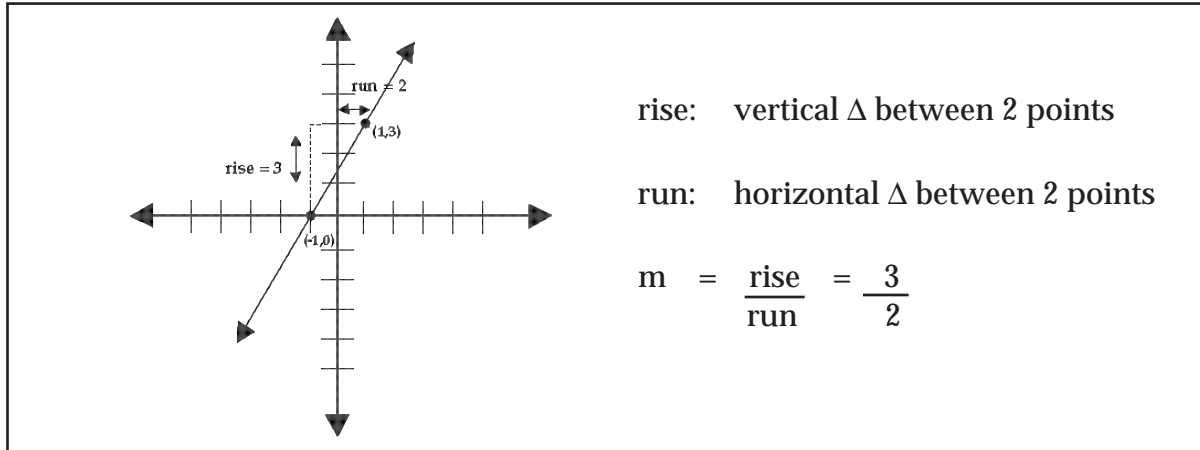


Make sure the students remember that lines are continuous, so the question of length becomes irrelevant. If they are ignoring the lengths of the ladder, you can discuss what is causing the steepness of the ladder to change in each situation. They should come up with the idea that the amount of vertical distance covered with respect to that covered by the horizontal distance is changing. This will head into the conclusion that the measure of steepness, slope, is actually a measurement of how fast height changes as length changes. At this point the following formula can be presented:

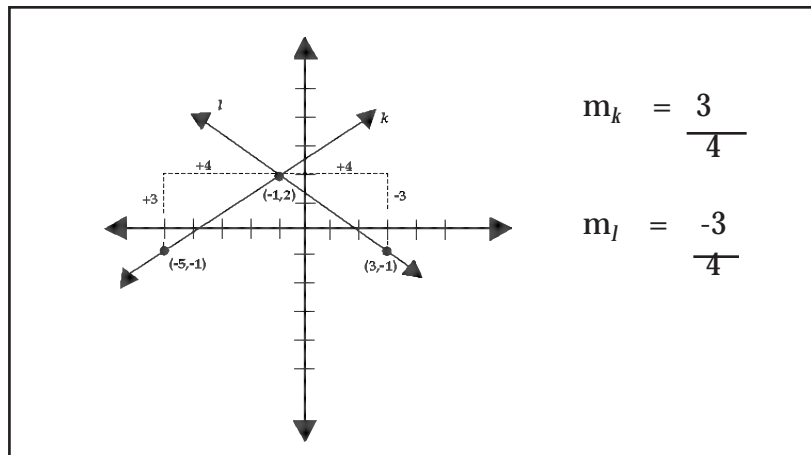
$$\text{slope } m = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{(Y_2 - Y_1)}{(x_2 - x_1)}$$

Δ : change in
m: slope

Go over an example such as the one below:



3. Next, ask how we might distinguish between two lines that have the same steepness, but different direction. Look at two lines with the same steepness, but opposite direction and calculate the slope, stressing the importance of reading changes from left to right or right to left, but not to mix and match.



This should lead the students to conclude that for / lines, both the rise and the run, working left to right are positive changes (or from right to left both are negative changes), so

$$\frac{\text{rise}}{\text{run}}$$

will be positive overall.

Such a line is called an increasing line.

Similarly, for \ lines, one change will be positive and the other will be negative, so the overall result will be negative. Such a line is called a decreasing line.

- Brainstorm for places where slopes are important. Ask how the value of the slope of a particular line, say a ski slope, tells how steep it really is. Sketch various lines and ask the students to indicate which line has the greatest slope and which has the least slope. A quick calculation of the slopes will confirm their hypotheses.
- Now, return to the ladder scenario. Ask the students what other than the slope must be fixed to give the specific placement of the ladder. Their conclusion should be either where you place the bottom or the top of the ladder.

Discuss where, if the ladder were in fact a line on a coordinate grid, these points would occur on the graph. The students should be able to identify them as being the points where the graph crosses the x and y axes. Some discussion can be had on the number of x and y- intercepts that a linear equation might have. If they decide that any linear equation must have one x and one y- intercept, you may wish to have them consider a vertical or horizontal line (eg. $y = 3$, or $x = 5$, etc.)

- Have each student sketch a line and identify the coordinates of the x- and y- intercepts of their line. Their results should be recorded as ordered pairs on a class chart like the one below:

<u>x- int</u>	<u>y -int</u>
(,)	(,)
(,)	(,)

Analysis of this chart should lead the students to conclude that an x-intercept occurs when $y = 0$ and that a y-intercept occurs when $x = 0$. Do a few examples of calculating these values from an equation.

$$\begin{aligned} &\text{eg. } 3x - 2y = 1 \\ \text{x-int: let } y &= 0 \\ &3x - 2(0) = 1 \\ &3x - 1 \\ &x = 1/3 \quad (1/3, 0) \\ \text{y-int: let } x &= 0 \\ &3(0) - 2y = 1 \\ &-2y = 1 \\ &y = -1/2 \quad (0, -1/2) \end{aligned}$$

- Have the students do a set of practice questions from pages 171 & 172 in *Mathematics: Principles and Process* (10). (Questions 1-5) and page 161 (Questions 1 and 2)

CELS:

- Creative and Critical Thinking
- Communication

Evaluation:

- Rating scale on positive participation0 - 5
- Homework completion rating scale0 - 2
- Slope and intercepts quiz6

Time Line:

- 1 - 2 hours

• SUB-UNIT 3: CHARACTERISTICS OF A LINE

LESSON 4: WHAT'S IN THE EQUATION

Overview of Topics:

1. Slope and y-intercept values and the slope- y- intercept form of a linear equation.
2. Slope, x-intercept, and y-intercept values from the standard form of a linear equation. (extension for entire class, or for the more mathematically confident students).

Student Required Materials:

- Graph paper
- *Mathematics 10*

Teacher Required Materials:

- Large class table for data collection

Procedures/Activities:

1. Assign each student in the class an equation written in standard form ($Ax + By = C$). Be sure to give each student an equation within their ability range. Ask each student to isolate the y in their equation (solve for y) so that their equation is in the form $y = mx + b$. You may need to do 2 or 3 examples of this for them. The results, and the original equation should then be recorded on the class table.

	$Ax + By = C$	$y = mx + b$	y-intercept	slope
eg.	$4x - 2y = 6$	$y = 2x - 3$	(0, -3)	2

2. Next, have the students graph their relation using a table of values, and record the results on the class table.
3. Once all of the data has been collected, have the students discuss the results in small groups, and then have the groups report back to the class as a whole. The conclusion that should be reached is that the slope- y-intercept form of a linear equation ($y = mx + b$) clearly gives both the slope and the y-intercept of the line. (m is the slope, and b is the y-intercept). As an option to this process, the students may have been in small groups for the entire procedure, developed a table of data, and then presented their results to the rest of the class.
4. (Optional) The preceding conclusions can be extended by having the students complete another chart like the one below:

$Ax + By = C$	x-int	y-int	slope	$-A/B$	C/A	C/B
eg. $4x - 2y = 6$	(3/2,0)	(0,-3)	2	2	3/2	-3

In this case, the students will find that while an equation is written in standard form, the x-intercept has a value of C/A , the y-intercept has a value of C/B , and the slope has a value of $-A/B$.

5. Assign the students some practice problems, such as page 236 in *Mathematics 10* (Questions 1 and 2)

CELS:

- Critical and Creative Thinking
- Personal and Social Values and Skills

Evaluation:

- Rating scale on positive participation 0 - 5
- Homework completion rating scale 0 - 2
- What's in the equation of a line quiz 4

Time Line:

- 1 - 2 hours

• SUB-UNIT 3: CHARACTERISTICS OF A LINE

LESSON 5: SLOPE, INTERCEPTS AND THE GRAPH OF A LINE

Overview of Topics:

1. Graphing from the equation of a line using the x and y-intercepts, the slope and an ordered pair, and from the slope- y-intercept form of the equation.

Student Required Materials:

- Graph paper
- Mathematics: Principles and Process (10)

Teacher Required Materials:

- Large grid for class to view

Procedures/Activities:

1. Ask students how many points you really need to know in order to be able to sketch a line. Once they have realized that the answer is two, ask them to plot the line going through the points (5,3), and (-4,1), or any such pairing of points. You may also want them to sketch the line containing the points (7,-3) and the origin. (To review the term origin).
2. Next, have the students review the concept of slope. Have them recall how slope was initially determined. Ask them what a slope of $\frac{2}{3}$ tells you. Eventually, they should suggest that between any two points, a rise of two will result in a run of 3, or that a rise of -2 (going down) will result in a run of -3 (going backwards).

Have the students consider the following question. On your fifth visit to Mount Scary, your favourite ski slope, you took measurements of the steepness of the mountain, which is renowned for its constant slope. You found that for every six meters of vertical change there was a horizontal change of 3 meters.

- a) What is the slope of Mount Scary?
- b) How could you accurately sketch the mountain?

(This should lead to a discuss of topics such as scale factors, and the selection of a point of origin — preferably the base of the mountain, but it could be elsewhere, such as the camp sight or a fire tower. Eventually, the question of how to use the slope to plot the graph should arise. Have the students recall how they actually obtained a value for slope at the beginning of this sub-unit to help come up with a method to graph the mountain.) Do a few more examples such as:

Graph the line that:

- a) has a slope of $-\frac{5}{2}$ and passes through (6,2)
- b) has a slope of 4 and passes through (-4,8)
- c) has a slope of $-\frac{1}{3}$ and has an x-intercept of 4
- d) has a slope of -1 and has a y-intercept of -3
- e) has a slope of $2\frac{1}{3}$ and passes through the origin.

Encourage the students to try different sign combinations on the rise and run values as well as not only seeing whole number slopes as being over 1, but also as rationals with other denominators.

3. Finally, ask the students what they could do to graph an equation such as $2x - 3y = 6$ without creating a table of values. Question the students on what kinds of information the equation of a line tells us, and how it might be used to obtain the graph of the equation.
 4. Assign a few practice problems, such as page 236 in *Mathematics: Principles and Process* (10) (8-14)
-

CELS:

- Communication
- Creative and Critical Thinking

Evaluation:

- Rating scale on positive participation0 - 5
- Homework completion rating scale0 - 2
- Graphing without a table quiz..... 8

Time Line:

- 1 hour

• SUB-UNIT 3: CHARACTERISTICS OF A LINE

LESSON 6: SPECIAL LINE CASES

Overview of Topics:

1. Slopes of parallel and perpendicular lines
2. Slopes of vertical and horizontal lines

Student Required Materials:

Graph paper

Mathematics: Principles and Process (10)

Teacher Required Materials:

Completed Special Lines form (see appendix) for each group in the class for both parallel and perpendicular lines.

Procedures/Activities:

1. Have the students work in small groups of three or four. Give each group a Special Lines form with equations of parallel lines to complete. Once the groups are done (maximum 15 minutes for completion of graph and discussion), discuss the results as a group. The students should quickly recognize the relationship between parallel lines and their slopes.
2. Repeat the above procedure using equations of perpendicular lines.
3. Divide the class in half. Give one half of the class $y = \#$ equations and the other half $x = \#$ equations (one per student). Have the students from each half report their findings: What does the graph look like, and what is its slope. A discussion of undefinedness and infinity will likely occur naturally.
4. Assign the students practice questions such as pages 183-185 in *Mathematics: Principles and Process* (10) (questions 6-9, 23).

CELS:

- Personal and Social Values and Skills
- Creative and Critical Thinking
- Communication
- Independent Learning

Evaluation:

- Rating scale on positive participation0 - 5
- Homework completion rating scale0 - 2
- Special Lines Quiz5

Time Line:

- 1 hour

• SUB-UNIT 4: WRITING LINEAR EQUATIONS

LESSON 7: THE EQUATION FROM SLOPE AND Y-INTERCEPT

Overview of Topic:

1. Writing the equation of a line where the slope and the y-intercept are known.

Student Required Materials:

- *Mathematics 10*
- *Mathematics: Principles and Process (10)*
- Graph paper

Teacher Required Materials:

- A large grid to graph on.

Procedures/Activities:

1. On the large grid, sketch a graph of a line. Have the students identify the slope, x-intercept, and y-intercept and then ask them how they could convert this information into an equation for the line. Usually the students are very quick to realize that all they need to do is substitute the slope in for m and the y-intercept in for b in the slope- y-intercept form of a linear equation. ($y = mx + b$).
2. In pairs, have each student sketch a line, and then derive the equation. Their partner should then be given the equation to plot. They should then compare the two graphs to make sure that they are the same.
3. Assign some practice questions such as page 193 in *Mathematics: Principles and Process (10)* (question 2) or page 236 in *Mathematics 10* (questions 2 and 3).

CELS:

- Creative and Critical Thinking
- Personal and Social Values and Skills

Evaluation:

- Rating scale on positive participation0 - 5
- Homework completion rating scale0 - 2
- Writing Equations — Part I quiz2

Time Line:

- 1 hour

• SUB-UNIT 4: WRITING LINEAR EQUATIONS

LESSON 8: SLOPE AND A POINT

Overview of Topics:

1. Recognizing the points on a graph as values that satisfy the equation of the graph.
2. Writing the equation of a line given the slope of the line and any point on the line.

Student Required Materials:

- *Mathematics 10*
- *Mathematics: Principles and Process (10)*

Teacher Required Materials:

- A large graph of a linear equation such as $y = x - 3$.

Procedures/Activities:

1. Have the students consider the graph and the equation that is on the large grid. Pick a value for x and then find the value for y from the equation. Then check if the point that you have is in agreement with the graph. Next, pick a point on the graph and ask the students how they might show that the point satisfies the equation.
2. Have the students do a few questions where they need to verify that a point satisfies the equation of a line. For example, questions 1, 3, 4, and 5 on page 142 in *Mathematics: Principles and Process (10)* can be used. (Note that some of #5 include inequalities, but they may prove to be an interesting extension.)
3. Discuss the following question as a class: If the point $(-1, -4)$ satisfies the equation $y = 2x + b$, where x and y are the variables, what does b have to equal. Once you have established b , ask the students how you can now get the complete equation.
4. Have the students do a few questions on page 143 in *Mathematics: Principles and Process (10)* (questions 9-11).
5. Now, discuss the following question. The plans for a wheelchair ramp have been drawn up on a grid. How could you determine the equation of the line representing the ramp? Once the students have been able to answer the question, have them review the steps of finding the equation of a line when given the slope and any point.
6. Have the students do a few questions on page 242 in *Mathematics 10* (questions 1 and 2).

CELS:

- Creative and Critical Thinking

Evaluation:

- Rating scale on positive participation0 - 5
- Homework completion rating scale0 - 2
- Writing Equations — Part II quiz.....5

Time Line:

- 1 - 2 hours

• SUB-UNIT 4: WRITING THE EQUATION OF A LINE

LESSON 9: TWO POINTS

Overview of Topics:

1. Finding the equation of a line when given two points on the line.

Student Required Materials:

- *Mathematics 10*

Procedures/Activities:

1. Ask the students: What can you find if you know two points on a line. Once they have identified slope, have the students form groups of two. Each pair is to pick two points and find the slope of the line containing those two points. Then have each student choose one of the two points and with the slope, derive the equation of the line from that information as they did in the last lesson. Have the students compare their results with each other, form a hypothesis about what they notice, and then repeat the process with another set of points. Once that is completed, they should have concluded that given two points on a line, they need only to find the slope using the two points and then select either point and the slope to obtain the equation of the line.
 2. Have the students do practice questions such as page 243 in *Mathematics 10* (question 6).
-

CELS:

- Personal and Social Values and Skills
- Creative and Critical Thinking

Evaluation:

- Rating scale on positive participation0 - 5
- Homework completion rating scale0 - 2
- Writing Equations — Part III quiz.....4

Time Line:

- 1 hour

• SUB-UNIT 5: DIRECT AND PARTIAL VARIATION

LESSON 10: DIRECT VARIATION

Overview of Topics:

1. Identifying real-life situations involving direction variation.
2. Solve proportions used in direct variations.
3. Solve word problems involving direct variations.

Student Required Materials:

- Graph paper
- *Mathematics 10*
- *Mathematics: Principles and Process (10)*

Teacher Required Materials:

- Large grid for graphing on

Procedures/Activities:

1. Use didactic questioning to obtain a student generated definition of direct variation. eg. What does variation mean to you? What do you think direct variation would mean? Where would you find examples of direct variation? In discussing the final answer, write their answers using the variation symbol (\propto)

eg. wages \propto time

2. With the examples the students have generated, have them generate a table of values for each and then plot them. Discuss what common graphical properties they are noticing and then write the equations. The result should be that they get an equation very similar to the slope- y-intercept form, but always with a y-intercept of 0.
3. Have the students compare the initial variation statement ($w \propto t$) to the resulting equation ($w = mt$). They should be picking up upon the constant value being the slope of the line. Define that value to also be the constant of variation because as the slope it tells us the constant relationship between the two variables.
4. Have the students consider the following problem. Earlier this year, your parents purchased 2250 L of diesel for the bulk tank on the farm at a cost of \$749.25. Lately, your parents have not been too impressed with all of your cruising around town, so they have decided to charge you the bulk price for all of the diesel that you use. If you put 28 L in your truck, how much will you owe your parents? (Prices for September 6, 1993).

Get the students to try to solve the problem by viewing it as a direct variation. They should recognize the constant of variation (value/volume) as the cost per litre. Remind the students that since the points (2250, \$749.25) and (28, ?) belong to the same variation, then the constant of variation must be the same, which means that a proportion can be set up and solved.

value \propto volume

$$v = k l$$

$$\frac{v}{l} = k$$

$$\frac{749.25}{225028} = k = \frac{x}{28}$$

$$x = \frac{28 (749.25)}{2250} = \$9.32$$

v: value
k: constant of variation
l: litres

5. Have the students try some of the problems on page 227 in *Mathematics 10* (questions 5-8, 9 extends into squared variations if time permits) or page 272 - 272 in *Mathematics: Principles and Process (10)*.
 6. As an added practice, each student can be required to create a word problem involving direct variation. The problems can then be exchanged and used as a review.
-

CELS:

- Creative and Critical Thinking
- Communication

Evaluation:

- Rating scale on positive participation0 - 5
- Homework completion rating scale.....0 - 2
- Created problem: originality/applicability5
solving problem.....5

Time Line:

- 1 - 2 hours

• SUB-UNIT 5: DIRECT AND PARTIAL VARIATION

LESSON 11: PARTIAL VARIATION

Overview of Topics

- Identifying and working with partial variations.
- Using partial variations to solve word problems.

Student Required Materials:

- Graph paper
- *Mathematics: Principles and Process (10)*

Teacher Required Materials:

- Large graphing grid

Procedures/Activities:

1. Introduce the topic of partial variation by brainstorming for what partial means and what in fact partial variation is.
2. Have the students consider the following question: When ordering grad jackets in grade 11, the cost for a jacket involves a fixed price for the jacket alone, and then a variable cost for all lettering that is to be added. From one company, the cost of a leather grad jacket is \$235 plus \$0.80 per letter that is to be added to the sleeve. Write an equation that calculates the cost of a jacket with any amount of lettering on it.

Encourage the students to develop a table of values and to plot it. From there, obtaining the equation should be straightforward.

3. Next, have the students compare the equation for the partial variation with those that they obtained for problems involving direct variation. They should notice that the constant term that is added on to the direct variation form is the y-intercept of the graph and that its value is the fixed value of the question. From this discussion, they should conclude that a partial variation's equation will have the form of:

$$y = mx + b \text{ where } y \text{ and } x \text{ are the variables}$$

m is the constant of variation
b is the fixed value

4. Have the students try some of the problems on pages 286-287 in *Mathematics: Principles and Process (10)*.
5. As a review, each student could be asked to create a partial variation problem.

CELS:

- Creative and Critical Thinking

Evaluation:

- Rating scale on positive participation.....0 - 5
- Homework completion rating scale.....0 - 2
- Created problem: originality/applicability.....5
solving of problem.....5

Time Line:

- 1 - 2 hours

• SUB-UNIT 6: ARITHMETIC SEQUENCES & SERIES

LESSON 12: BASICS OF ARITHMETIC SEQUENCES

Overview of Topics:

1. Identification and defining of arithmetic sequences.
2. Determining the n th term of an arithmetic sequence.

Student Required Materials:

- Graph paper
- *Mathematics 12*, Brendan Kelly, Paul Atkinson, Bob Alexander. Don Mills, ON: Addison-Wesley, 1991.
- *Math Is/6*, Frank Ebos, Mary Crowley, Bob Tuck. Scarborough, ON: Nelson Canada, 1982

Procedures/Activities:

1. Ask the students for examples of sequences in their every day lives. Encourage the students to think of non-mathematically based examples such as the sequence of steps to putting on a pair of shoes, or for baking a cake. Next encourage them to think up examples of mathematical sequences. Define each value in a sequence (or step) as a term.
2. Once the students are familiar with the concept of a sequence, carry out the following concept attainment or something similar. Remind the students that they should not say what they think the rule is until they are asked.

YES EXAMPLES	NO EXAMPLES
1,2,3,4,...	1,2,4,8,...
-3,2,7,...	-3,-3,-3,...
$\frac{1}{2}, \frac{1}{4}, 0, -\frac{1}{4}, \dots$	$\frac{2}{3}, -2, 6, -18, \dots$
8, 2, -4, -10,...	2,4,7,11,...

Next, without the students saying what they think the rule is, ask them if the following are yes or no examples:

- a) 5.3,5.9,6.5,...
- b) x, x_2, x_3, \dots
- c) $x-1, x+1, x+3, \dots$
- d) $2x + 5, 4x+5, 6x+5$

Finally, ask the students to identify the characteristics of an arithmetic sequence:

- adding between terms only
- difference remains the same for all terms
- the difference must be non-zero

3. Ask the students to give the first five terms of the arithmetic sequence with a first term (called a) of 7 and a common difference of -2. As they give you the terms, write them in a column and in a row form as below:

term #	term value
1	7
2	5
3	3
4	1
5	-1

7, 5, 3, 1, -1, ...

Ask the students if there is some way that a graph of the information contained in the sequence could be given. The result should be a graph, with the term # as the independent variable and the term value as the dependent variable, that is linear. Ask the students to compare the results with those of partial variations. They should see that the first term of the sequence is like the fixed value of the partial variation and that the common difference between the terms is acting in the same way that the variable constant did in partial variations. The students should then be able to write the equation for the sequence:

$$v = -2n + 7 \quad \begin{array}{l} v \text{ is the value of the term} \\ n \text{ is the term number} \end{array}$$

Next, ask the students how they could use this equation to determine the value of the 78th term.

$$\begin{aligned} v &= -2(78) + 7 \\ &= -156 + 7 \\ &= -149 \end{aligned}$$

As well, ask them how they could use this equation to determine what term number has a value of -95.

$$\begin{aligned} -95 &= -2n + 7 \\ -102 &= -2n \\ 51 &= n \end{aligned}$$

4. Develop the general equation for the terms in an arithmetic sequence, using t_n to represent the value of the nth term, n to represent the term number, a to represent the value of the first term, and d to represent the common difference.

$$t_n = a + (n-1)d$$

5. Ask the students for an example in real life that an arithmetic sequence would occur.
eg. A test in one of your classes is out of 100%. Short answer questions account for 60% of the mark, while the remaining 40% is for 20 multiple choice questions. When you wrote the test, you were confident that you had answered all of the short answer question correctly, but you are unsure of how you did on the multiple choice questions. What possible marks could you get on the test?

This generates the following sequence of numbers:

60, 62, 64, 66,...

You could then ask the students what the values for a and d are and to write the general equation for a term in the sequence. Ask the students what a specific term in the sequence represents. You may also want to ask the students questions like: How many of the multiple choice questions did you get correct if your exam mark was 84? Is it possible to have a mark of 87? What is your mark if you got 35 of the multiple choice questions correct?

5. Have the students do some practice problems on pages 376-378 in *Mathematics 12* or on pages 440-445 in *Math Is/6*.

CELS:

- Creative and Critical Thinking
- Communication

Evaluation:

- Rating scale on positive participation.....0 - 5
- Homework completion rating scale.....0 - 2
- Arithmetic sequences quiz.....7

Time Line:

- 2 - 3 hours

• SUB-UNIT 6: ARITHMETIC SEQUENCES & SERIES

LESSON 13: ARITHMETIC MEANS

Overview of Topics:

1. Definition of an arithmetic mean.
2. Calculation of arithmetic means.

Student Required Materials:

- *Math Is/6*

Procedures/Activities:

1. Have the students consider the following problem. You just received back a test on which there is only a percentage given. You got 8 questions right and got a mark of 67%, while your friend got 11 questions right and got a mark of 79%. After going over the exam, you feel that you actually have one question right that was marked wrong. What mark can you expect to have if you are correct about the marking error? (Assume that all questions are worth the same amount and that they are marked on a completely right, or completely wrong scale).

Although there are variety of ways to solve this problem, try to encourage the students to set the question up as an arithmetic sequence:

$$67, _, _, 79$$

Then, $a = 67$, $n = 4$, and $t_n = 92$ and d can be solved for.

$$\begin{aligned}t_n &= a + (n-1) d \\79 &= 67 + 3d \\12 &= 3d \\4 &= d\end{aligned}$$

So, with a common difference of 4 marks, you can expect a mark of $67 + 4 = 71$.

2. Define the terms between any two given terms in an arithmetic sequence as being the arithmetic means of the numbers. Indicate that there can be any number of means specified. A brief discussion of what a single mean is will lead into the ideas of average.
3. Give the students some sample questions on page 442 in *Math Is/6* (questions 18 - 20).

CELS:

- Creative and Critical Thinking

Evaluation:

- Rating scale on positive participation 0 - 5
- Homework completion rating scale0 - 2
- Arithmetic means quiz.....4

Time Line:

- 1/2 - 1 hour

• SUB-UNIT 6: ARITHMETIC SEQUENCES & SERIES

LESSON 14: ARITHMETIC SERIES

Overview of Topics:

1. Finding the sum of an arithmetic series.

Student Required Materials:

- *Math Is/6*

Procedures/Activities:

- A useful, general introduction is the “Gauss” problem.

1. Define a series as being the sum of the terms in a sequence (finite number of terms). So if a sequence is:

3,7,11,15,19,23
the series is $3+7+11+15+19+23$
for which the value is 78.

2. Give the formulae for the sum of an arithmetic series:

$$S_n = \frac{n(2a + (n-1)d)}{2}$$

$$S_n = \frac{n(a + t_n)}{2}$$

where S_n is the sum of n terms, and a, d, n , and t_n are as defined before.

If there is time, or just for the more capable students, the development of these formulae is beneficial as it gives the students some insight into a mathematical “trick” that they have not seen before.

$$S_n = [a] + [a+d] + [a+2d] + \dots [a + (n-1)d] \quad n \text{ terms}$$

$$\text{Write } S_n \text{ backwards. } S_n = [a + (n-1)d] + [a + (n-2)d] + [a + (n-3)d] + \dots + a \quad n \text{ terms}$$

$$2S_n = [2a + (n-1)d] + [2a + (n-1)d] + [2a + (n-1)d] + \dots + [2a + (n-1)d] \quad n \text{ terms}$$

$$2S_n = n [2a + (n-1)d]$$

$$S_n = \frac{n [2a + (n-1)d]}{2}$$

3. Have the students consider the following problem. When you were born, your Godmother decided to start a trust fund for you. On your first birthday she put \$25 in the account, on your second birthday she put \$35 in the account, on your third birthday she put \$45, and so on (\$10 increases per year). How much money will she have put aside by the time you are 16?

4. Have the students do some questions from pages 461-462 in *Math Is/6*.
 5. Extension: Look at summation notation, and index changing.
-

CELS:

- Communication
- Creative and Critical Thinking

Evaluation:

- Rating scale on positive participation0 - 5
- Homework completion rating scale0 - 2
- Arithmetic series quiz3

Time Line:

- 1 - 2 hours

• SUB-UNIT 7: SCATTERPLOTS

LESSON 15: SCATTERPLOTS

Overview of Topics:

1. Collecting data for and preparing scatterplots.
2. Analyzing the scatterplots.
3. Construction of a 'best fit' line by sight.

Student Required Materials:

- Graph paper

Teacher Required Materials:

- Timer
- Large graphing grid

Procedures/Activities:

1. Divide students into 3 groups. If possible, it is best to work in a hallway or gymnasium. Assign the groups as slow, medium, and fast walkers. Have the students walk for a specific time, about 2 minutes, at their designated speed. It is their job to keep track of the number of steps they have taken. Once they are done, have them take their pulse. This data should then be recorded on a table like the one below:

# of steps	pulse
------------	-------

Remember to discuss independent and dependent variables!

Plot the data on the large grid and then discuss the results - ask them for conclusions and interpretation. Eventually this should lead to the idea of the results being linear and obtaining a 'best fit' line by sight. Some mention should be given to the fact that there are statistical methods for determining an exact line of best fit, but for now the line will only be approximated. Have the students locate a line that they feel is the 'best fit' line and then determine its equation. Discuss how the equation could be then used in the interpretation of the data.

2. Have the students form groups of 3 or 4. Each group is to select an experiment that will generate a linear scatterplot. (For examples see *Algebra Experiments I: Exploring Linear Functions* — anything with a basically constant rate of change will do). Once their experiment has met your approval they are to carry it out, record the necessary data, plot the data, give a written analysis of the results, and draw in the best fit line and determine its equation. Once this is completed, the group is to make up a visual display of their results and present it to the class.

3. To review scatterplots, the following synectic presentation could be used. Ask students:

- How is a scatterplot like a shopping mall?
- How is a scatterplot unlike a shopping mall?
- How would it feel to be someone attempting to get from one point in the mall to another?
- How would it feel to be a point in a scatterplot? (Include both the points on the 'best fit' line and not on the 'best fit' line).
- What else could you compare a scatterplot to?
- Why?

For the last question, you may have them turn in their answers for marks, or just for assessment of their understanding.

CELS:

- Personal and Social Values and Skills
- Communication
- Creative and Critical Thinking
- Numeracy

Evaluation:

- Rating scale on positive participation..... 0 - 5
- Scatterplots Project.....25
- Scatteplot Analogy5

Time Line:

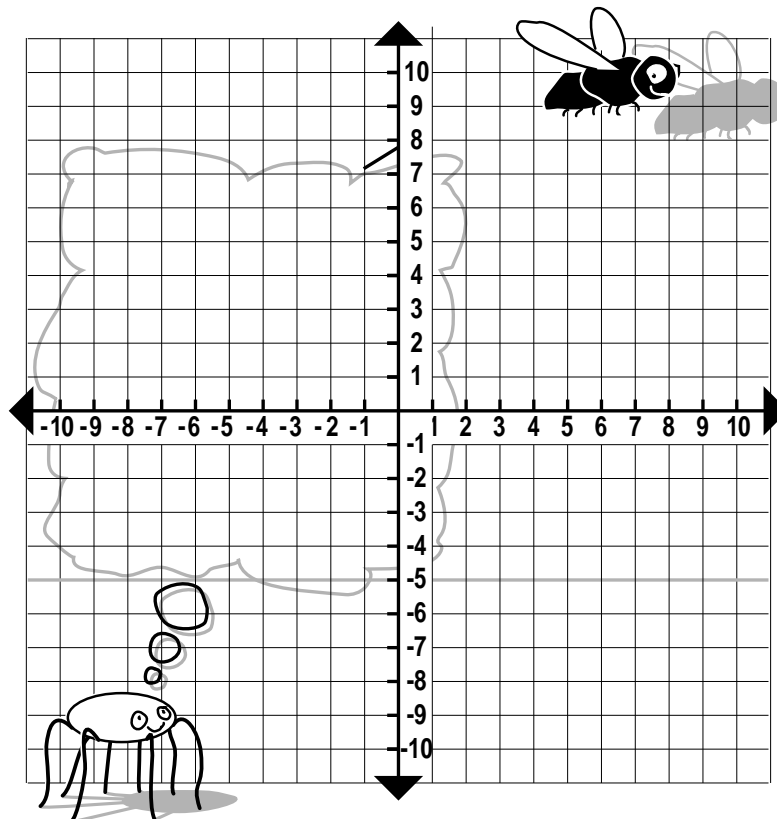
- 3 hours maximum

• APPENDIX A

THE SPIDER AND THE FLY

Plot the points in each set of ordered pairs. Use straight lines to connect each point in a set with the previous ones in the set, but do not join the sets together.

1. $\{(-7,1), (-7,3), (-6,3), (-6,1)\}$
2. $\{(-7,-2)\}$
3. $\{(-3,6), (-4,6), (-4,4), (-3,4)\}$
4. $\{(-6,-1), (-6,-4)\}$
5. $\{-5,3), (-3,3)\}$
6. $\{(-10,6), (-10,4), (-9,4)\}$
7. $\{(-7,-3)\}$
8. $\{(-6,4), (-6,6), (-5,4), (-5,6)\}$
9. $\{(-7,2), (-6,2)\}$
10. $\{(-2,5), (-1,5)\}$
11. $\{(-9,-1), (-9,-4), (-8,-4), (-8,-3), (-9,-3)\}$
12. $\{(-8,6), (-8,4), (-7,4), (-7,6)\}$
13. $\{(-4,3), (-4,1)\}$
14. $\{(-1,4), (-1,6)\}$
15. $\{(-5,-1), (-5,-4), (-4,-4), (-4,-1), (-5,-1)\}$
16. $\{(-2,6), (-2,4)\}$



CREATIVE PLOTTING

Task:

1. Using a minimum of 10 points (maximum 25), plot a picture, message or design on a piece of graph paper. Make sure that you have points in each of the four quadrants.
2. Make a list of the points that you have plotted as ordered pairs. If the order of the points is important, be sure to indicate it along with the list of points.
3. Have someone else in the class take your list of points and plot them on a separate piece of graph paper.

Turn in:

1. Your original plotting of the points.
2. Your list of points and instructions.
3. The plotting of your points done by another student.

Marking Scheme:

Originality/Creativity:5
Accuracy of listed points and instructions:.....10
Accuracy of plotting10

SNAP TO IT!

In your groups of three, each person must assume one of the following roles:

- snapper
- counter
- timer

If your group only has two people in it, the snapper and counter can be combined into one job.

What to do...

1. The timer will time out periods of time starting with 5 seconds, and progressing to 30 seconds at 5 second intervals. During the timings, the snapper is to snap their fingers (at as constant a speed as is possible) as many times in the time period as they can. It is the counter's job to count the number of finger snaps that are done in each timed period and to record the results in the table below:

TIME INTERVAL (SECONDS)	NUMBER OF SNAPS
5	
10	
15	
20	
25	
30	

2. Decide which variable, time or number, is the independent variable and then graph the result. Although there are no negative values involved, you should still show all four quadrants, but they do not need to be the same size.
3. Turn in your graph. Be sure to clearly identify your group members on the graph.

SPECIAL LINES

Your group's equations are:

1. _____
2. _____
3. _____
4. _____

Task:

1. Each group member selects one of the above equations. Analyze the equation for its x-intercept, y-intercept, and slope and record the data in the chart below.
2. Each member should then graph their equation on a sheet of graph paper. All graphs should be put on the same sheet.

Questions:

1. What do you notice about the lines?
2. What do you notice about the data in the chart?
3. What hypothesis can you make from your results?
4. Report back to the class as a whole.

EQUATION	X-INT	Y-INT	SLOPE

SCATTERPLOT PROJECT

Task:

1. In your group of 3 or 4 people, select an experiment whose data should generate basically linear data and have it approved by the teacher.
2. Carry out the experiment. Carefully collect and record the data from the experiment.
3. Analyze the data by:
 - graphing
 - sketching the 'best fit' line
 - finding the equation of the 'best fit' line
 - writing up conclusions or hypotheses that you feel can be made from your data and an explanation why
4. Create a visual presentation for your experiment. It should clearly represent your data and should explain the results that you were able to obtain. Neatness and creativity of the presentation are also important.

Turn in:

- data
- analysis
- visual presentation

Marks:

Originality of Experiment.....	3
Recorded data	2
Written analysis of experiment	10
Clarity of visual presentation	5
Neatness/Creativity	5

APPENDIX B

NAME _____

PLOTTING EQUATIONS BY TABLES OF VALUES QUIZ

Create a table of values for the relation $y = 3x + 2$ (3 points minimum), then graph the results on the graph paper below).

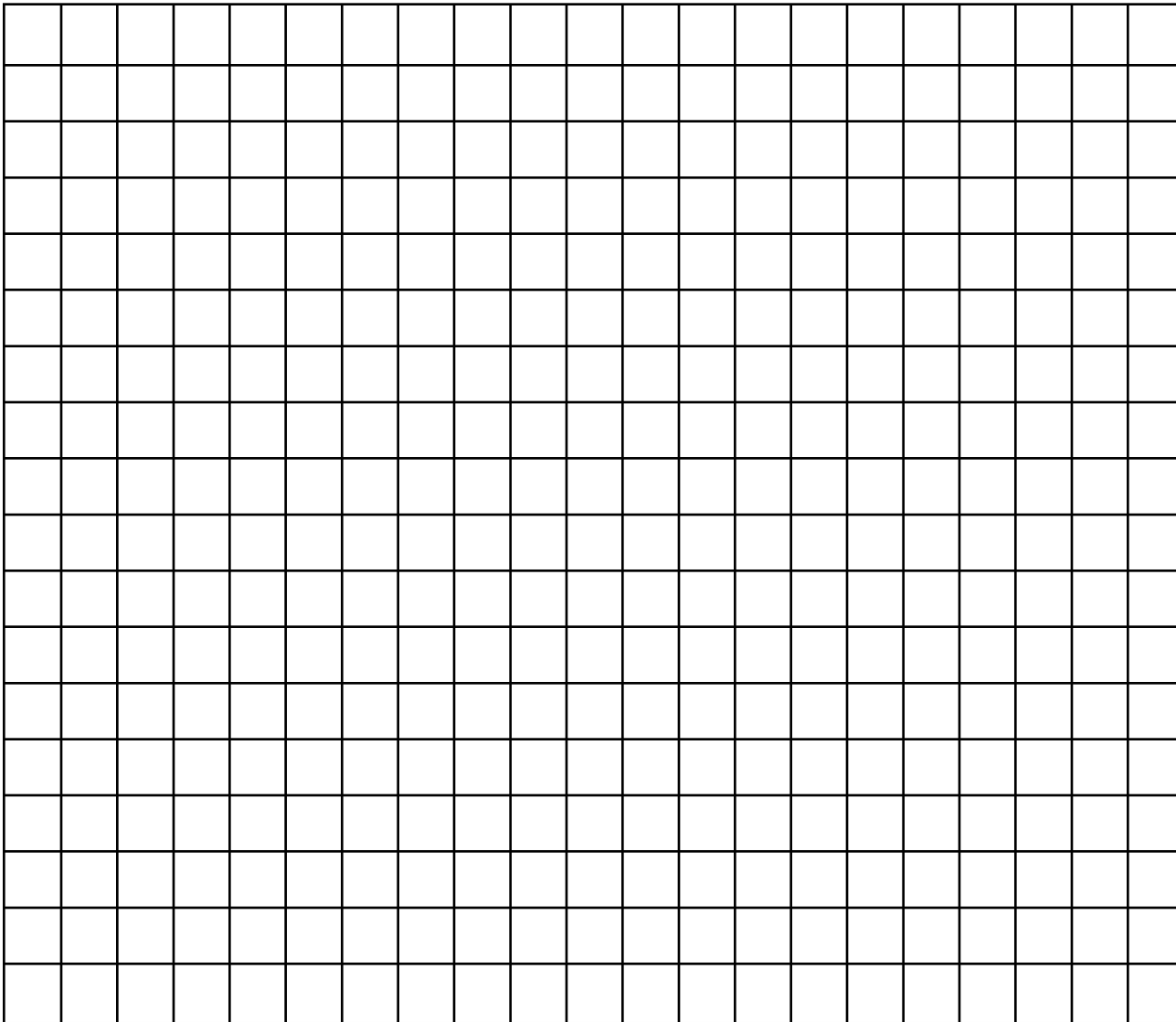
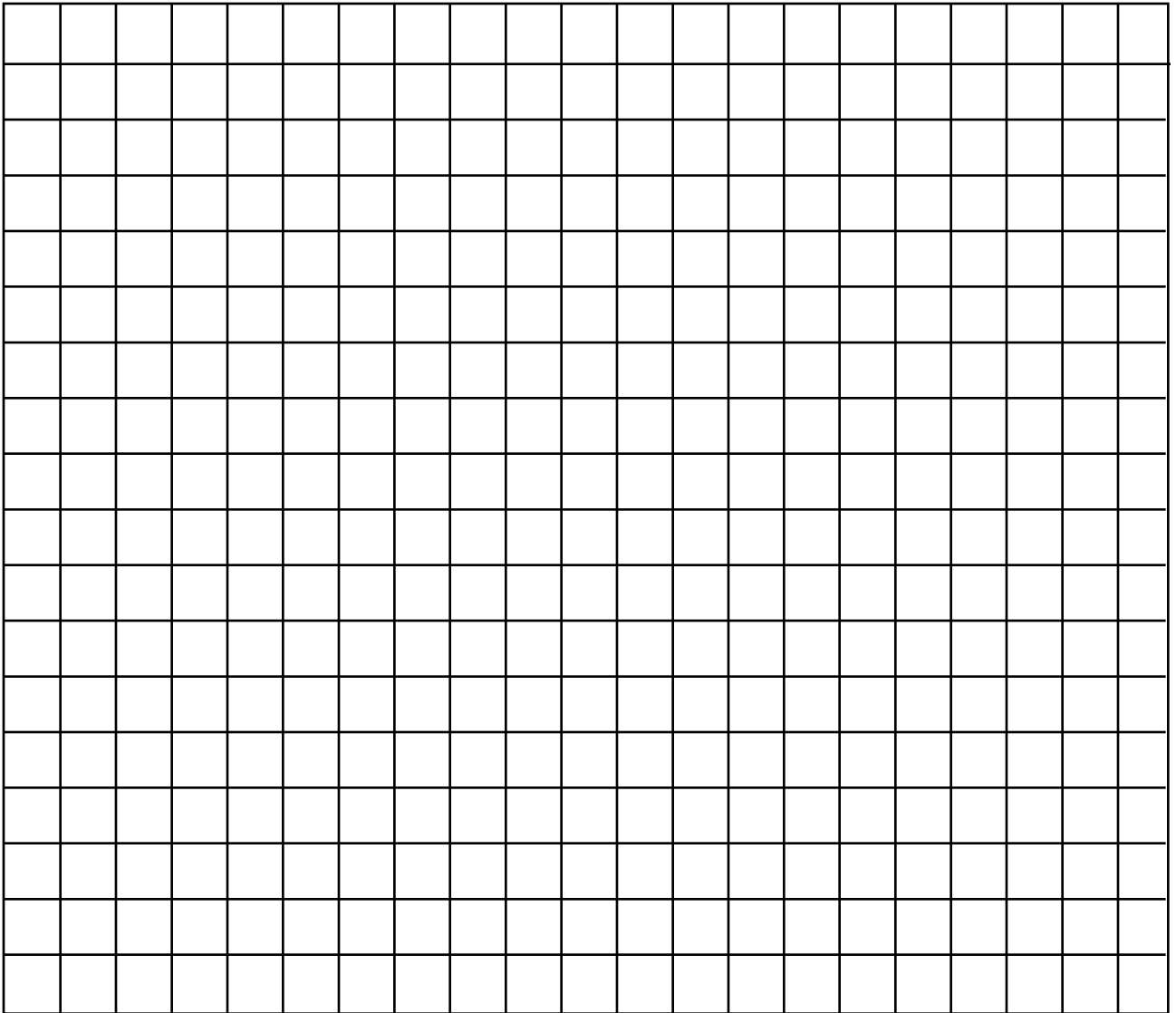


Table of Values	3
Graph	<u>4</u>
Total	7

NAME _____

SLOPE AND INTERCEPTS QUIZ

Find the x-intercept, y-intercept and slope of the line defined by the equation $y = 2x + 4$.

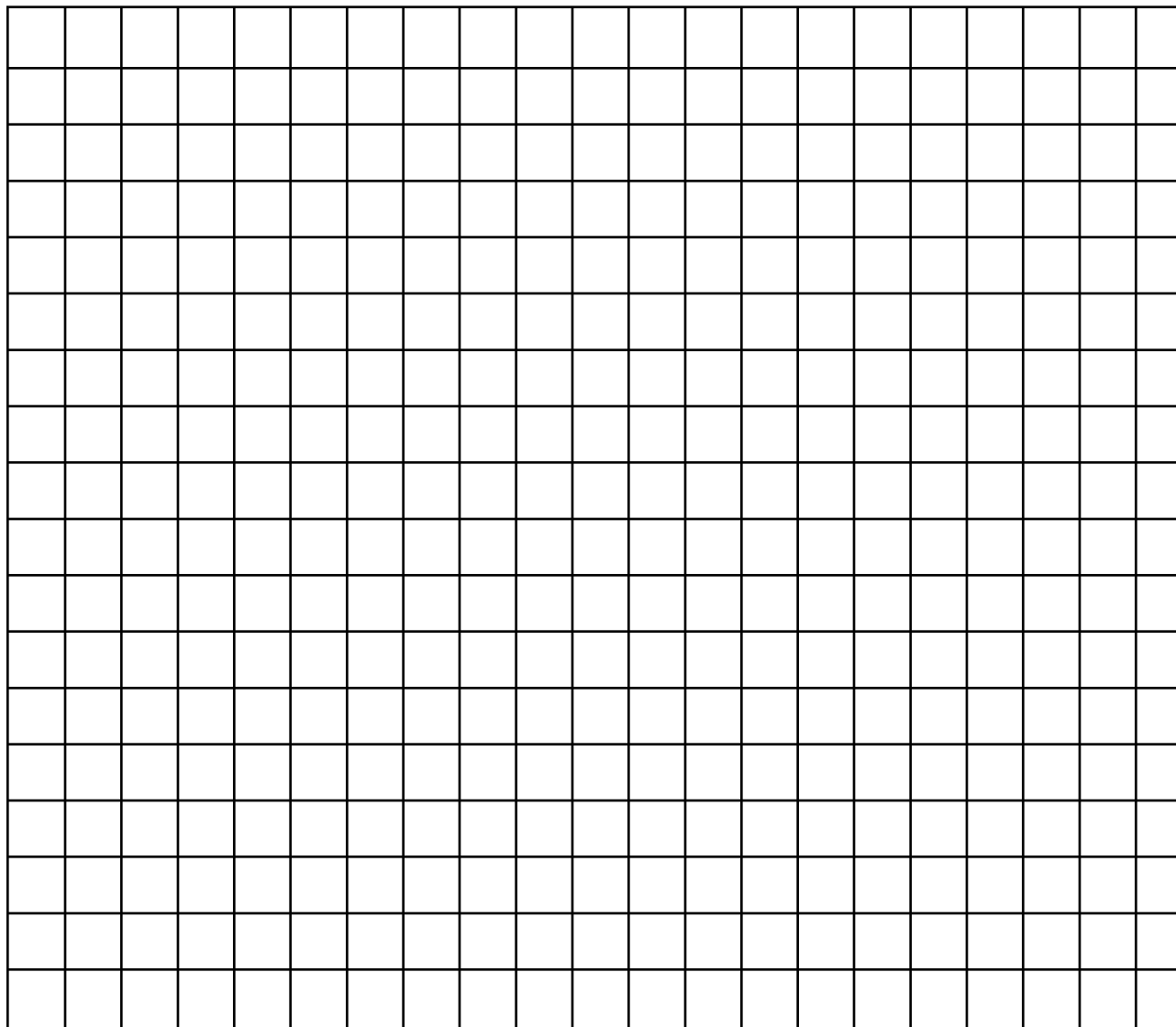


x-intercept:	/2
y-intercept:	/2
slope:	/2
total:	<hr/> /6

NAME _____

WHAT'S IN THE EQUATION OF A LINE QUIZ

Write the equation $3x - 5y = 2$ in slope- y-intercept form. Then identify the slope and the y-intercept. Graph.

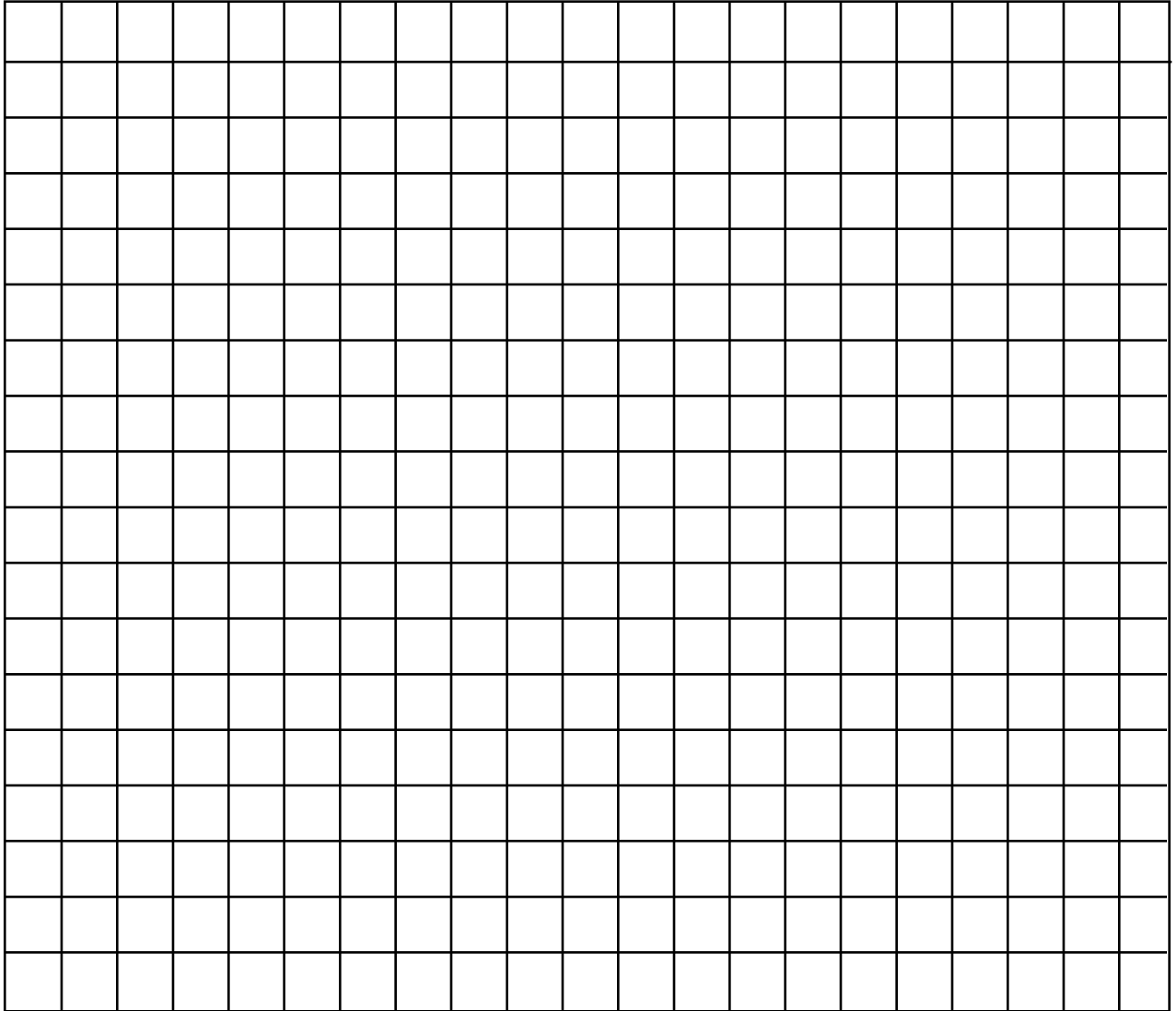


slope-y--intercept form	2
slope:	1
y-intercept	1
graph	<u>1</u>
	5

NAME _____

GRAPHING WITHOUT A TABLE

Graph the equation $y = 7x + 5$ on the grid provided below without using a table of values. Be sure to indicate how you are doing this.



reasoning	4
graph	<u>4</u>
	8

NAME _____

SPECIAL LINES QUIZ

1. What is the slope of a line which is parallel to the line $2x + y = 3$?

2

2. What is the slope of a line perpendicular to the line $y = -3x + 4$?

2

3. What type of line is given by the equation $x = -6$?

1

NAME _____

WRITING EQUATIONS — PART I

Write the equation of a line with a slope of -9 and a y-intercept of 0.

2

NAME _____

WRITING EQUATIONS — PART II

1. Does the point (7, -1) satisfy the equation $2x + 3y = 10$? Give reasons for your answer.

2

2. Find the equations of a line that has a slope of $-2/3$ and contains the point (0,5).

3

5

NAME _____

WRITING EQUATIONS — PART III

Write the equation of a line that passes through the points $(-1, 2)$ and $(6, -3)$.

4

NAME _____

ARITHMETIC SEQUENCES

1. What is the 58th term in the following sequence:

-3, 2, 7, ...

3

2. What term number has a value of -70 in the sequence

2, -2, -4, ...

3

3. Give an example of a sequence which is not arithmetic.

1

7

NAME _____

ARITHMETIC MEANS

Find the three arithmetic means for -7 and 39.

4

NAME _____

ARITHMETIC SERIES

Find the sum of the first 30 terms of the series

$$1+4+7+ \dots$$

Be sure to show your work!

3