The differential equation describing current through an LR circuit is

$$L\frac{dI}{dt} + RI = V(t),$$

where $I(t)$ is the current, R the resistance, L the inductance and $V(t)$ is the applied voltage supplied by the power supply.

1. **Use the integrating factor method** to determine the current as a function of time when the voltage is $V(t) = V_0(1 - \exp(-t/T))$, where V_0 and T are constants. Take the current to be initially zero.
2. Determine the time taken for the current to be 10% of the final value.

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$$L\frac{dI}{dt} + RI = V(t)$$

$$\frac{dI}{dt} + \frac{RI}{L} = \frac{V(t)}{L}$$

$$p(t) = \exp\left(\int \frac{R}{L} dt\right) = \exp\left(\frac{R}{L}t\right)$$

$$\frac{d}{dt}\left(\exp\left(\frac{R}{L}t\right)I\right) = \frac{1}{L}\exp\left(\frac{R}{L}t\right)V(t)$$

$$\int \frac{d}{dt}\left(\exp\left(\frac{R}{L}t\right)I\right) = \int \frac{1}{L}\exp\left(\frac{R}{L}t\right)V(t)\cdot dt + C$$

$$\exp\left(\frac{R}{L}t\right)I = \int\left[\exp\left(\frac{R}{L}t\right)\cdot\frac{V(t)}{L}\right]\cdot dt + C$$

$$\begin{aligned} I(t) &= \exp\left(-\frac{R}{L}t\right)\cdot\left[\int \exp\left(\frac{R}{L}t\right)\cdot\frac{V(t)}{L}\cdot dt + C\right] \\ &= \exp\left(-\frac{R}{L}t\right)\cdot\left[\int \exp\left(\frac{R}{L}t\right)\cdot\frac{V_0\left(1-\exp(-t/T)\right)}{L}\cdot dt + C\right] \end{aligned}$$

$$
\begin{aligned}
&= \exp\left(-\frac{R}{L}t\right)\cdot\left[\frac{V_0\left(1-\exp\left(-t/T\right)\right)}{L}\cdot\frac{L}{R}\exp\left(\frac{R}{L}t\right)+C\right] \\
&= \frac{V_0\left(1-\exp\left(-t/T\right)\right)}{R}+C\cdot\exp\left(-\frac{R}{L}t\right)
\end{aligned}
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