M.C.Escher: Mathematics and Visual Arts

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(A more detailed version of this unit, containing supplementary ideas and activities, worksheets, evaluation instruments, a magazine article and a bibliography will be inserted in the mathematics curriculum guide for middle years which will be available in June 1996.)

Integration

Integration is a term that is appearing more and more frequently in the professional teaching literature. In its simplest form, integration means making connections or links.

Integrated teaching brings the following advantages:

- integration helps students to make connections since the links between various concepts, skills and attitudes are established;
- since students make connections, transferring what they've learned to new situations is more readily accomplished;
- integration fosters a deeper understanding of concepts, since linked ideas allow students to review, to test hypotheses and to assimilate concepts in a more efficient way;
- integration also allows students to gain an overview, since learning situations are placed in a larger context;
- students can then make connections between the learning situations and experiences in everyday life, and learning thus becomes more relevant and more motivating;
- since unnecessary overlaps are avoided, integration saves time (an important factor for classes that are multigrades and for designated schools);
- teachers who plan integrated units with colleagues receive the benefit of their expertise and knowledge, as well.

For teaching according to the new mathematics curriculum, several ways of integrating can be identified:

- simple integration, by which the teacher makes links between two or three topics in mathematics: the link between the decimal system and our system of currency, for example;
- integration of a mathematics topic with a life skill: an example is the link between problem solving in mathematics and working cooperatively;
- integration within the mathematics program: problem solving, data management, numbers and operations, geometry and measurement, ratio and proportion, and algebra are the six strands of the mathematics curriculum guide at the middle years level. The intent is to teach the six strands in an integrated fashion; that is, a lesson or a series of lessons would address the specific learning objectives of more than one strand at a time. For example, problem solving is integrated into all the other strands, and geometry can be used for learning concepts of area and volume as well;
- for another kind of integration, the teacher plans lessons in similar topics or objectives in several compulsory study areas and teaches them at the same time during the school year but in different class periods; in this way symmetry can be taught in mathematics as well as in art education, and measurement in mathematics as well as in natural sciences. The links between them are made explicit for the students;
- at a more complex level of integration, the class periods for mathematics and art education simply become one period during which symmetry is taught, the class periods for mathematics and natural sciences become one during which measurement is taught, and the class periods for mathematics and social studies become one period during which data management is taught;
- integration can also make links between all the compulsory study areas, through use of a theme. The teacher uses a conceptual diagram (also known as a spider web) to map out the links between the study areas in a visual form; the theme chosen should be easy to apply to all the study areas. This theme could be derived from one that has already been used in another study area. This is a form of

integration that is enjoying increasing popularity among teachers;

- another form of integration uses one of the Common Essential Learnings or one of the objectives for a CEL as a theme in all the study areas; for example, all the required areas of study can be linked through the common theme of technological literacy;
- integration is also achieved through the tools for symbol making, conceptualization and expression, that is, through languages. This is the model for immersion, and it's the basic principle of communication and the introduction to numeracy (CEL). Some subjects concern themselves with the study of concrete matters (natural and human sciences, hygiene). Others are languages (French or English, mathematics and fine arts). "The concrete could not be symbolized without language and, conversely, language could not exist without some content to symbolize. When this view is taken, it becomes possible to teach languages in close correlation with the aspects of the concrete world whose symbolization they allow." (Tardif, 1992)

In this unit, we see integration of two subject-languages (mathematics and visual arts) with the concrete (the art of M.C. Escher).

This unit is intended for use at the grade 7 level, but it could easily be modified and adapted so as to respond to the needs of students at other levels.

There is a large variety of levels and ways of achieving integration.It is recommended that teachers experiment with a number of methods for carrying out integration.

Summary of the Unit

After they've studied examples of tiling encountered in everyday life, the students explore tiling from the perspective of mathematics and visual arts through studying the works of M.C. Escher.

Instructional Approaches They are:

- cooperative learning;
- guided visualization;
- classification;
- manipulation of objects;
- demonstration;
- problem solving;
- data collection;
- discussion;
- computer-assisted learning.

Resources and Materials

- Pattern blocks, logic blocks and/or tangrams.
- Protractors.
- Works of art by M.C. Escher.
- Examples of paving and tiling found in magazines, books or the environment: patterns used in flooring, patios, wallpaper, fabrics, quilts, stained glass windows, mosaics, ornamentation from different cultures, works of art.

Vocabulary and Structures

In the course of this unit, the student will understand and use:

- the vocabulary and the structures associated with mathematical activities: covering a surface, paving an area, mosaics, paving, tiling, tessellation, two dimensional shapes, polygons, triangles, quadrilaterals, trapezoids, rectangles, squares, pentagons, hexagons, octagons, congruency, symmetry relative to a line, symmetry by rotation, sliding, turning, flipping, angles;

- the vocabulary and the structures associated with visual arts: evenly filling an area, rhythmical repetition of a pattern, decorative art, ornamentation, paradox, metamorphosis, an impossible world, drawing from observation.

Introduction

Present the students with **examples** of paving, tiling or tessellations.

Examine a number of these paved surfaces and point out which shapes are used in creating them.

(The following examples can be taken from magazines or books or be found in the environment: patterns used in kitchen or bathroom floors, patios, honeycombs, wallpaper, fabrics, quilts, stained glass windows, mosaics, ornamentation from different cultures, works of art.)

By studying these techniques you can **develop a definition** of paving, tiling or tessellation.

(Tiling, paving or tessellation is defined as covering a surface or an area by means of polygons that are placed in such a way as to leave no space between them and to have no overlap of the polygons.)

(Another way to develop a definition of paving is to use the learning strategy called **concept attainment**, and to show students examples and non examples of paving. See the pamphlet *That's a Yes! Concept Attainment,* which appears in the collection titled *Instructional Strategies Series.*)

Give each student a pattern block (the yellow hexagon, the red trapezoid, the green triangle, the orange square or the blue rhombus) and ask the students to cover a surface or pave an area (a rectangle measuring 20 cm by 25 cm) so as to produce a tessellation. The teacher can demonstrate at the same time, using the overhead projector, by placing the tile in the middle of the area, tracing its outline, and placing it in another spot, and continue to work in this way (from the middle out to the sides) until the whole area has been covered (without leaving any space between the tiles).

Exploration

Is it possible to **develop criteria** for determining whether a shape can be used to tessellate or not?

Activity:

Give the students (who are working in pairs or in groups) a large number of two dimensional shapes and ask them to divide these shapes into two groups according to the following criteria:

- shapes that can be used to cover the area;
- shapes that cannot be used to cover the area.

Students can be asked to produce a hypothesis regarding this **classification** (criteria of classification), to verify it, and to compare the results with their hypotheses.

These results can be shared with the rest of the groups.

You can pin up the examples of paving and tiling in the classroom and ask the students to add further examples.

You can keep the criteria and the results of this experiment posted on the bulletin board in order to be able to come back to them as the unit goes on.

Here is a sample of the shapes that can be used: the circle, the oval, a variety of polygons (triangles, quadrilaterals, pentagons, hexagons, octagons, etc.). Irregular polygons should also be presented. The different forms can be numbered to make discussion easier.

It's an appropriate juncture at which to encourage teachers to begin drawing up a list of mathematical and other terms that are specific to this unit. The list can be pinned up and added to throughout the unit. The students can refer to it. Making an illustrated dictionary can be an activity they carry out along with their projects.

Activity: Students select one or more shapes and **cover the area** (completely cover the surface).

Students use their imagination to draw and colour the design.

Extension

Present to the class a number of **works by the artist M.C. Escher** and make mention of the fact that he was fascinated by regularity and mathematical structure, by continuity and the infinite, and by the latent conflict in each image. Discuss with them the sources of his inspiration and what had influenced him.

Escher used geometrical shapes which he would transform so as to generate other interesting shapes. He would then draw and/or colour his piece. Show the students how to **transform a square** using translation and rotation.

The students can then cover the surface of the area using the work of M.C. Escher as a model.

For further details on how to proceed, refer to the article titled *The Art of Mosaics* in the Arithmetic Teacher of March 1990.

Reflection

Ask the students to **write down their personal impressions** of the activities in this unit, the relationship that exists between mathematics and visual arts, and such creative strategies as feeling the excitement of discovery, looking for the various possibilities afforded by the representation of an idea, and bringing out the contradictions.

Reinvestment

Present a scenario for the whole class or ask the students to choose between the following options:

Scenario 1:

Ask the students to perform transformations starting with **shapes other than the square**, such as the triangle, the hexagon, etc.

Draw and/or colour them.

Scenario 2:

Use the **software program** *Tessellmania* to explore different transformations and ways to cover a surface.

Scenario 3:

Make a **kaleidoscopic construction**. Find three square mirrors measuring approximately 30 cm per side. Glue them together to form a corner. Cut out shapes that have been traced on wood, plastic, stiff cardboard or some other material. Arrange a composition on the mirror's surface. Draw attention to the fact that the figures are completed by their reflection in the mirror. For example, a semicircle becomes a circle. Glue the arrangement to the surface with a suitable type of glue.

Scenario 4:

Mount a **project for a Math Fair**. You can plan to invite parents and other classes from the school.

Possible projects:

- describe and illustrate how to transform shapes;
- assemble a collection of linoleum tiles, patio bricks, wallpaper samples, etc.; describe the shapes used and the transformations they have undergone;
- draw up a chronological summary of Escher's life story;
- write and illustrate a dictionary of the terms that are specific to this unit;
- write about what was learned in the course of this unit;
- prepare a report on another artist's work;
- study the concept of visual paradox as it shows up in the work of various artists, such as, for example, Salvador Dali, André Breton, Max Ernst, René Magritte, Miro, Hans Arp, André Masson. Look at fantasy, dreams, eccentric imagery, optical illusions, and surrealistic perceptions in two- and three-dimensional works (consult *Les fiches du musée. Pour mieux comprendre les créateurs et les mouvements modernes et contemporains / The Museum Index. For a Better Understanding of Creators and Modern and Contemporary Movements,* National Museum of Modern Art, 1992-93, as well as encyclopedias and dictionaries).

Other Suggestions for Adapting the Unit

- You can study the aspects of perimeter and area of the different shapes. What happens to the perimeter and the area when a shape is transformed? (The perimeter changes, but the area remains constant.)
- You can use the software programs *Tessellmania* and *EscherSketch.*
    - One activity would be to ask the student to use the computer to alter a shape, cover the surface, print out his results and explain, orally or in writing, the kinds of transformations that have taken place (translations, rotations, flips, or their combination).
    - The computer provides a way for students who have problems with motor skills to explore the subject and increase their knowledge of tessellations.
    - For another activity, the student is given a shape that has already been transformed and is asked to interpret this shape in a creative way using the drawing functions of the software program; she can add eyes, mouths, etc.
    - For another activity, the student selects a piece by Escher, identifies the original shape and, with the help of the computer program, tries to transform the shape so as to reproduce the artist's design.
    - The student can be asked to create a tessellation that would be suitable for a floor, a T-shirt, wrapping paper, etc.
- Explore kaleidocycles (three-dimensional shapes that can turn). Build a kaleidocycle using tetrahedrons. Each student should have 8 copies of the tetrahedron pattern. Ask the students to draw and/or colour them. The tetrahedrons are folded individually. They are then pasted up one after another to form a circle. The circle can be turned, while the tetrahedrons undergo a rotation.
- Explore platonic solids. They can be identified and constructed. There are 5 platonic solids: the regular tetrahedron, the regular hexahedron, the regular octahedron, the regular dodecahedron, and the regular icosahedron. These solids are polyhedrons in which all the sides are regular congruent polygons. Thus the tetrahedron has 4 sides (congruent triangles); the hexahedron has 6 sides (congruent squares); the octahedron has 8 sides (congruent triangles); the dodecahedron has 12 sides (congruent pentagons); and the icosahedron has 20 sides (congruent triangles).
- Find a number of different paving styles that can be created with rectangles (in proportions of 2:1 and/or 3:1). Talk about what these proportions represent. What other proportions could be used? Ask the students to present examples of pavements.
- What happens when you work with irregular pentagons? Can you pave a surface or create a mosaic with these irregular pentagons? Can you use any irregular pentagon to create a mosaic?
- Escher also worked with ideas of limits and infinity. Find specific pieces that have limits. Find a piece in which he's dealing with the infinite (the Möbius strip). Why does the Möbius strip represent infinity? Make a Möbius strip yourselves to find out the answer. Here is how to make a Möbius strip: take a band of paper about 35 cm long by 2-3 cm wide by the two ends; turn just one of the ends a

half-turn and join the two ends with a piece of adhesive tape. You will then have created a circle with a half-turn in it. Ask the students to find a starting point on the strip and put the tip of their pencil on this point; without lifting their pencil, they then draw a line along the middle of the strip, through its entire length. What happens? After having drawn a line on both sides of the strip, the pencil comes back to the starting point. You can also say that this is a piece of paper that has only one side!

- Find pieces by Escher in which the artist works with two dimensions and the three dimensions.
- What is the smallest number of colours required to colour an area so that no two adjoining pieces are the same colour? You can begin this investigation by dividing a surface into 2, 4, 6, etc. parts with straight lines.
- Interpret a fantasy theme by creating a symmetrical drawing. Create a fantasy version of a deck of cards. For example, choose the king, the queen, the jack or the joker. Use a subject of your choice as the initial visual for the portrait (head and torso). Work with an 18" x 24" sheet of paper. Draw the portrait on one half of the sheet. On the other half, make a copy of the drawing with tracing paper and finish it off with acrylics or poster paints.
- Which letters of the alphabet could be used to pave a surface? Can you use your initial? both your initials? Try it (you can distort the letters and transform them a little).
- Ask the students to perform the following exercise: to make a contour drawing of their hand or foot. Next, bring in some object in which the students can see themselves, such as a mirror, a Christmas tree ornament, etc. Each student holds the mirror or the ornament in one hand and studies the hand and its reflection. Distortion should be encouraged. Find the works of M.C. Escher in which distortion is present.
- You can explore visual arts from the perspective of distortion and the surreal:
    - Create a surreal landscape inside a shoebox using a combination of different techniques and two- and three-dimensional materials, such as, for example, photographs, found objects, and shapes that have been sculpted, moulded or painted , which you arrange together inside the shoebox.
    - Construct a Neo-Dadaist collage from various images that have been cut out of magazines, newspapers, photocopies or archival photographs. Assemble them by carefully cutting and pasting them.
    - Compose a bizarre story about an experience or an observation. Write down the story of a real event or experience. Then write a story recounting a dream that seemed especially bizarre or surreal. Illustrate these stories using the medium of your choice (refer to *The Book of Images*, Éditions Atlas, page 60).
    - Create a surrealist bust. Take a styrofoam head and transform it by adding bits of cloth, leather, buttons, string, mechanical gadgets, bottle corks, wool, paint, etc.
    - Create a surrealist monument. Make up a surreal event that would rival those in *The Book of Records*, or use *The Book of Records* as a source for ideas, and create a trophy, a medal or a banner in recognition of this act of bravery.
    - Explore the process of colour photocopying as a medium of artistic expression. Colour photocopying is a printing process based on three colours (yellow, blue and red). The advantage of this process is that you can easily change the desired colour. You can also make a black and white print. First make a collage using a selection of images you've clipped from magazines and posters. Arrange the images on a piece of construction paper and glue them on. Add some hand drawn elements. Use the colour photocopier to make prints of your collage.
    - Create an optical illusion by combining graphics, movement and strobe lighting. Cut a 30 cm circle out of stiff cardboard. Draw a series of concentric circles 2.5 to 5 cm apart. Use acrylics to paint designs within the different segments of the disk. Mount the disc on the transmission of a small motor and attach it to a tabletop. Use a lighting dimmer switch to control the motor's speed. Illuminate the disc with a strobe light. You'll see that the shapes within the concentric circles will show up simultaneously or separately depending upon the motor's speed.
- Perform an integration with other study areas:
    - draw up a chronological summary of Escher's life;
    - locate Escher in history: what other famous people were alive at the same time as Escher, what important events happened in the history of the world or of Canada during Escher's

    lifetime;
    - Escher had a great deal of trouble with mathematics when he was at school, and yet his works include much mathematical content. How can that be? Ask the students to write about the question. Another famous person who had trouble with mathematics was Albert Einstein;
    - Then what is mathematics?
    - Why do you think they call Escher the poet of the impossible?
- Give the students a research assignment to find works of art from different cultures that demonstrate tessellation or symmetry, etc.
- You can study optical illusions.
